Analysis of the Vertical Accuracy of the CTAS Trajectory Prediction Process

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Introduction

Since 1989, the National Aeronautics and Space Administration (NASA) has been developing elements of the Center-Terminal Radar Control (TRACON) Automation System (CTAS). CTAS provides a broad array of Decision Support Tools (DST's) that assist controllers in performing a variety of air traffic control (ATC) functions. A key function of CTAS is to model aircraft trajectories to allow predictions of future aircraft positions to assist controllers in efficiently and effectively managing air traffic within their jurisdiction. A key issue in CTAS development is the accuracy of the CTAS modeling function.

Two flight demonstrations were conducted (Phase I in October 1992 and Phase II in September 1994) to evaluate CTAS accuracy during the en route arrival phase of flight. The Transport System Research Vehicle (TSRV) Boeing 737 Model 100 aircraft, based at NASA Langley Research Center, flew a combined total of 57 arrival trajectories during these demonstrations.

The lateral and time accuracy from these demonstrations were analyzed and reported in papers and reports completed in the late 1990's, [1] and [2]. The research effort reported herein analyzes the vertical accuracy of the CTAS prediction resulting from the 1994 demonstration.

Background

Previous analysis and assessments have measured CTAS prediction errors of arrival time, navigation ("cross track"), and vertical profile [1]. Further analysis and assessments have identified and measured the contributions of the major sources of arrival time error [2].

The objective of this research effort is the identification and measurement of the major sources of error in CTAS predictions of the vertical component of aircraft trajectories (in the same vein as the accounting for the major sources of error in CTAS arrival time predictions [2]). The data set contains 25 runs, each of approximately 15 minutes duration with aircraft state as a function of time.

Tasking - Accounting of CTAS Vertical Profile Error

A key requirement is to define the metrics to best describe vertical profile error. At a minimum, these metrics will include the along-track error at the Top-Of-Descent (TOD) and/or the Bottom-Of-Descent (BOD) events. (For example, for the runs using the TSRV vertical navigation function (VNAV) to determine TOD, the along track error at the TOD should be reported.)

A second requirement is to perform analyses to identify and measure the sources of error in the vertical component of the CTAS trajectory predictions for all 25 TSRV field test runs. These runs consist of 6 descents (7 descents in one case) for each of four automation types and accompanying procedures).

A third requirement is to analyze the flight test data (on a flight-by-flight basis) to identify the contribution from each error source identified. The goal is to account for all errors within a residual of 100 ft. or less. Then combine results for common runs to form aggregate results as in [2]. It should be noted that during the analysis it was discovered that some of the altitude data required for the analysis had a resolution of 100 feet. Because of this, the goal for the residuals was relaxed to 200 feet.

Flight Demonstrations

Flight demonstrations were accomplished in two phases. Phase I occurred in October 1992 and focused on straight-path descents. Phase II occurred in September 1994 and evaluated CTAS trajectory prediction along a more complex arrival route that included a turn during the descent. Expanded capability aircraft systems (Flight Management Systems (FMS) with Lateral Navigation (LNAV) and VNAV functions were used in the demonstrations. In addition, four descent procedures and three speed schedules were used. The demonstrations also made use of lessons learned in the Phase I demonstrations.

Flight Procedures

The TSRV has two flight decks with different navigation capabilities. The forward flight deck contains conventional Very High Frequency Omni-directional Radio Range (VOR) and Distance Measuring Equipment (DME). The rear flight deck contains advanced technology equipment that contained capabilities that were similar to a state-of-the-art FMS. This equipment provided LNAV and VNAV capabilities similar to those in modern commercial airliners.

<u>Conventional Descents</u>: Descents using this "conventional" equipment are typically described as "open loop" with the pilot initiating descent at TOD by bringing the throttles to flight idle. Typically, the speed during the descent is controlled to some predefined Mach number, Calibrated Airspeed (CAS) value during the descent. The Mach/CAS schedule varies by airline flight operations policy and aircraft make and model. During the conventional descent, the vertical profile is typically not controlled. The aircraft is allowed to descend at rates that are characteristic of the aircraft's performance capabilities. However, if the flight crew believes the aircraft is not descending fast enough (or slow enough) to arrive at some downstream fix at the correct speed and altitude, the crew, at their discretion, may add speed brakes (spoiler deployment) to descend faster, or add thrust to descend slower at their discretion. Hence, even though this descent is largely "open loop," pilot discretion adds a degree of "closed loop" control.

<u>VNAV Descents:</u> With the VNAV functions available to the pilots in the rear flight deck of the TSRV, the aircraft can be flown on a precise, predefined vertical profile during the descent. The FMS uses information about the aircraft performance capabilities, weight, flight path and winds to define the vertical profile. During the descent, the aircraft's flight control system controls the aircraft speed, thrust and speed brake deployment to maintain the predefined vertical profile and speed schedule.

<u>Identification of TOD:</u> Three methods were used to define the TOD in the flight demonstrations. The first method was to have CTAS identify TOD as a specified DME distance from the Denver VOR/Tactical Air Navigation (VORTAC) facility. This information was then relayed from the CTAS site to the flight crew via radio prior to the descent. This method is referred to as CTAS TOD. The second method for identifying TOD was to use the point defined by the FMS VNAV trajectory prediction function. This method is referred to as FMS TOD. The third method of TOD identification was a range-altitude arc (RAA) method. This method makes use of simplified descent calculations to identify the TOD point. This method is referred to as the RAA method.

Descent Procedures Used in the Flight Demonstrations: Four descent procedures were used in the Phase II demonstrations. These are:

a. Conventional VOR/DME procedures with CTAS TOD flown from the forward flight deck
b. VNAV descent with FMS TOD flown from the rear flight deck
c. VNAV descent with CTAS TOD flown from the rear flight deck
d. VNAV descent with RAA TOD flown from the rear flight deck

When the "c" and "d" descent procedures were used, the aircraft began descent using the specified TOD procedure. When the aircraft intercepted the VNAV profile, the crew (or the flight control system) captured the VNAV profile and flew the VNAV profile from that point to the BOD.

Mach/CAS Speed Schedules

All descents began at Flight Level (FL) 330 and ended at 17,000 feet Mean Sea Level (MSL). In accordance with normal flight procedures, the barometric correction from 29.92 in of Mercury (Hg) to local barometric correction was applied as the aircraft approached FL 180. Three Mach/CAS schedules were used during the demonstrations. These were:

1. Medium-Speed Descent: If necessary, slow the aircraft to .73 Mach at FL 330, bring the throttles to flight-idle, descend at .73 Mach until the CAS increases to 280 knots, then descend at 280 knots until reaching BOD at 17,000 feet MSL. Level the aircraft at 17,000 feet MSL and slow the aircraft to 250 knots indicated airspeed (KIAS).

2. Slow-Speed Descent: Bring the throttles to flight-idle and slow the aircraft to a CAS of 240 knots at FL 330. Descend at 240 knots until reaching BOD at 17,000 feet MSL. Level the aircraft at 17,000 feet MSL.

3. High-Speed Descent: If necessary, slow the aircraft to .76 Mach at FL 330, bring the throttles to flight-idle, descend at .76 Mach until the CAS increases to 320 knots, then descend at 320 knots until reaching BOD at 17,000 feet MSL. Level the aircraft at 17,000 feet MSL and slow the aircraft to 250 KIAS.

CTAS Descent Procedures

The En Route Descent Advisor (EDA) tool of CTAS uses the knowledge that the desired end conditions (three-dimensional position, time, and speeds) at BOD are known. The unknowns are the necessary initial conditions at TOD to achieve those BOD conditions. To solve for the TOD conditions, EDA assumes certain aircraft performance parameters (thrust, drag, weight, and speed schedules) and atmospheric conditions (temperature profile, wind speed and direction) and integrates backward in time from the known conditions at BOD to solve for the unknown TOD conditions. These initial conditions are then known as CTAS TOD conditions.

During the 1994 demonstrations, The BOD end conditions were that the aircraft be at the metering fix at an altitude of 17,000 feet MSL at a specified speed (240 KIAS for the slow-speed descent, 250 KIAS for the medium-speed and high-speed descent). For each of the test descents, EDA used speed schedules similar to those used in the aircraft, which are described in the previous paragraphs.

Two notable situations occurred in the execution of the CTAS runs during the flight demonstrations. The first concerns the assumed weight of the aircraft during descent. The nominal weight used by CTAS for the Boeing 737-100 was intended to be 85,000 pounds for all descents. However, during many of the descents an assumed aircraft weight of 98,000 pounds was used. This was caused by a software interface problem with the Federal Aviation Administration's (FAA's) Air Route Traffic Control Center (ARTCC) computer that was not detected until after the conclusion of the demonstration. This error is noted in the analyses as "weight experimental error."

The second notable situation concerns the application of barometric correction in CTAS during the demonstrations. In accordance with FAA rules, flight procedures specify use of a standard barometric setting (29.92 inches of Hg) at altitudes above FL 180 (nominally 18,000 feet, MSL assuming a standard atmosphere) and a local barometric setting at altitudes below 18,000 feet. Because the demonstration involved descents from FL 330 to 17,000 feet, MSL, the flight crew and aircraft systems made the transition from uncorrected barometric altitude above FL 180 to corrected barometric altitude below 18,000 feet. The change in barometric setting at 18,000 feet MSL is included in the CTAS software and should have properly accounted for altitude transitions. However, due to a procedural error, the barometric correction was applied with the incorrect sign in all of the CTAS descents. This error is identified as "barometric correction error" in the analysis section of the report.

Sources of Vertical Error

There were two primary cases tested in the 1994 program: conventional descent procedures (Mach/CAS profiles at descent thrust initiated with varying cues), and VNAV guided descent procedures (also constrained to a Mach/CAS profile). The following error sources were identified as being quantifiable from the existing test data results.

- Vertical error induced by early or late initiation of the descent
- Vertical error induced by differences between the true airspeed of the aircraft and the true airspeed used in the CTAS model. These errors are due to a number of sources to include piloting errors in controlling to the intended Mach/CAS profile, atmospheric differences in temperature and lapse rate that affect Mach number and CAS conversions to true airspeed
- Performance degradation of the NASA TSRV Boeing 737-100 aircraft caused by modifications to the aircraft for research purposes
- Vertical errors induced by differences in descent performance as affected by aircraft weight. The error resulting from incorrectly-applied weight values used in some cases to develop the CTAS profile (referred to as 'experimental error') was considered separately from errors induced by differences in the assumed CTAS weight of 85,000 pounds and the actual weight of the aircraft during each descent
- Vertical deviation due to the effect of lateral navigation errors increasing path length
- Vertical error induced by differences between the aircraft and CTAS ground speed resulting in elongation (or shortening) of the aircraft descent path
- Errors induced by wind gradients in the along track direction (wind gradients affect the overall energy of the aircraft)
- Errors caused by the application of thrust during the actual aircraft descent that were not accounted for in the CTAS descent
- Errors caused by the application of speed brakes (spoilers) during the actual aircraft descent that were not accounted for in the CTAS descent

- Vertical deviation due to errors in modeling ambient temperature along the descent path
- Errors induced by two barometric correction sources: 1) differences between aircraft and CTAS barometric corrections (referred to as barometric setting error), and 2) incorrectly applying barometric altimeter setting during development of the CTAS profile (referred to as ‘barometric correction experimental error’)

Analysis

Discussion of Vertical Error Sources

The error source convention used in the analysis is to subtract the actual aircraft quantity from the CTAS quantity. The vertical error model functions were developed in the following manner:

Total Altitude Error: Total altitude error is the difference between uncorrected aircraft barometric altitude and uncorrected CTAS barometric altitude (CTAS flight level times one hundred). Thus, if the aircraft is above the model profile at a given point along track, the error is positive. Units for the altitude error are in feet.

Descent Initiation Error: The magnitude of the effect of early or late descent initiation on vertical error is calculated from a determination, through analysis of the CTAS model profile, of the effect (in altitude) that such an along track difference would yield by shifting the CTAS profile by that amount. Initially, the altitude effect is zero because both the aircraft and the CTAS model are nominally at FL 330. Once the descent portion of the flight is reached, the altitude effect remains relatively uniform. If the actual aircraft descent initiation is late, then the aircraft is above the model profile, and the error is positive.

Speed Deviation Errors: Vertical errors result from deviations from the aircraft and CTAS true airspeed values during the descent. These deviations may be pilot-induced flight technical error in maintaining Mach/CAS descent profile or may be caused by differences in the atmosphere assumed by CTAS and that encountered by the aircraft. The sign of the resulting error is such that a positive speed deviation yields a steeper descent, and the resulting altitude error is negative.

Performance Degradation Error: Because the TSRV is a research aircraft, a number of changes have been made to the aircraft that increase drag. A discussion of this error is contained in [1] (Appendix A, page 39). This error has been quantified in terms of a percentage change in the quantity "thrust minus drag" during descent. This error was measured in [1] as varying linearly from –9 percent at FL 330 to –2 percent at 17000 feet MSL.

Aircraft Weight Deviation Errors: The effect of weight on descent gradient is such that a higher actual aircraft weight results in a smaller descent gradient (with altitudes above the CTAS nominal path), causing a positive altitude error. Two types of weight deviation errors were considered. The first type is an experimental error caused when a weight of 98000 pounds was erroneously used on some CTAS descents. The nominal aircraft weight for CTAS descents for the TSRV was intended to be 85000 pounds. This error only applies to those cases wherein the incorrect weight was used in the CTAS descent. The second type of weight deviation error is the deviation of the actual aircraft weight from the nominal weight of 85000 pounds. In most cases this deviation was less than 8000 pounds and contributed moderate errors to the vertical profile.

Wind Gradient Errors: Changes in the along-track winds (headwinds or tailwinds) encountered by the aircraft cause energy to be added or removed from the aircraft. The fundamental result is that a positive tailwind gradient (increasing tailwind) causes kinetic energy to be added to the aircraft and potential energy to be subtracted from the aircraft, which results in a lower altitude. Conversely, an increasing headwind (negative wind gradient) causes an increase in aircraft altitude. Thus, an aircraft wind gradient that is greater than the predicted CTAS wind gradient causes the aircraft to be below the CTAS profile, resulting in a negative altitude error.

Along-Track Error – Distance: Deviation of the aircraft from the nominal path typically causes the aircraft to fly a longer path than the CTAS path. If the aircraft path length exceeds the CTAS path length, the resulting error in altitude is negative (aircraft altitude is below CTAS altitude).

Along-Track Error – Speed: The effect of differences between the aircraft and CTAS ground speed along the flight path also cause differences in the path length similar to those described for the Along-Track Error – Distance. When the aircraft ground speed exceeds the CTAS ground speed, the resulting error in altitude is negative (aircraft altitude is below CTAS altitude).

Thrust Application and Spoiler Deployment Deviations: The addition of thrust (above flight idle) reduces the descent rate, which causes the aircraft profile to be above the CTAS profile. The deployment of spoilers (speed brakes) during descent increases the descent rate, which causes the aircraft profile to be below the CTAS profile.

Outside Air Temperature Model: The effect of temperature deviations from the CTAS nominal profile was assessed by deriving the effect of temperature deviation on true air speed (TAS) given that the flight is controlled to a specific CAS or mach number velocity. The sign of the effect is such that on a hotter day, TAS is higher, resulting in a steeper descent angle (altitude error is negative).

Barometric Setting Errors and Barometric Setting Experimental Errors: For the vertical profile analysis herein, there are two sources of barometric correction error considered. The barometric setting error (BSE) is the difference between the actual aircraft barometric setting and the barometric setting assumed by the CTAS model. The barometric correction experimental error (BCExpE) is the effect caused by the barometric correction being applied in the wrong sense during all of the CTAS runs during the flight demonstration.

Neither BSE nor BCExpE come into play until either the aircraft or CTAS model reaches their final descent altitude. Figure 1 shows the case for a high-pressure day where the aircraft descends an actual vertical distance of 16000 feet + aircraft barometric correction. Because of the experimental error, the CTAS model descends a vertical distance of 16000 feet – CTAS barometric correction. The BSE is the difference between the FMS correction and the CTAS correction (if it had been set with the correct sign). The BCExpE is the vertical difference between the CTAS altitude with the incorrect CTAS barometric setting and the CTAS altitude with the correct CTAS barometric setting. This value is equal to twice the vertical distance from the incorrect CTAS barometric setting to the standard (29.92 in Hg) barometric setting altitude. On a low-pressure day, the errors are a mirror image about the standard barometric setting (29.92) line. For the flight demonstrations, all of the descents experienced the high-pressure conditions as depicted in Figure 1.

In the test data, the aircraft barometric correction height is determined by subtracting the uncorrected altitude from the corrected altitude at the bottom of descent point. The CTAS barometric correction height is calculated by subtracting the MSL altitude at the bottom of descent (17000 feet) from the CTAS uncorrected altitude at the end of the descent. BSE is calculated by subtracting the CTAS height from the FMS height. BCExpE is calculated by doubling the CTAS barometric correction height.

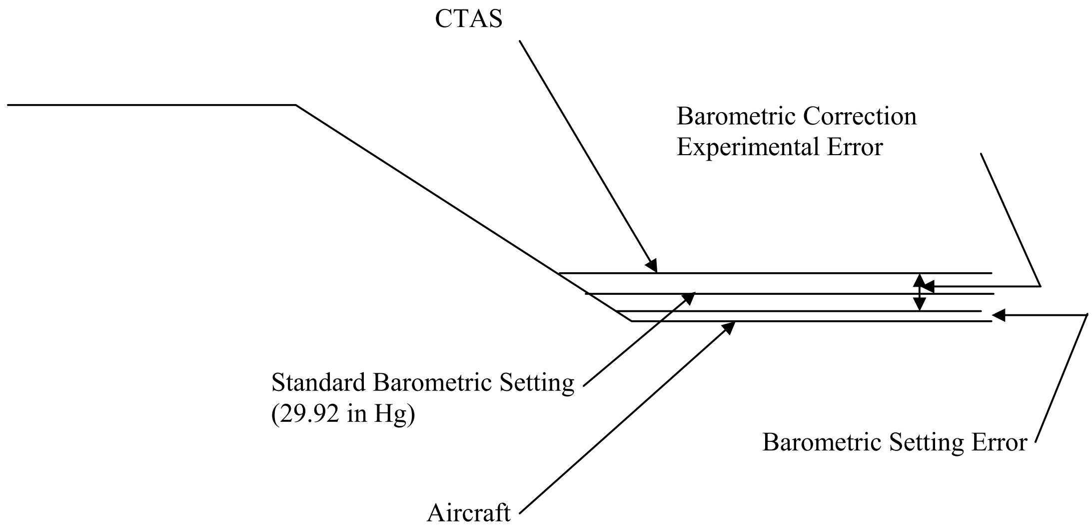


Figure1 Barometric Setting Error and Barometric Correction Experimental Error (high-pressure day)

Data Sources

Profile data were available four types of files – radar, Global Positioning System (GPS), abbreviated FMS files and complete FMS files. The radar and GPS files are outputs of NASA's Trajectory Analyzer tool. The radar files contained the following data:

> CTAS horizontal trajectory data at selected event-driven points in the trajectory (11 parameters)
>
> CTAS vertical trajectory data at selected event-driven points in the trajectory (19 parameters)
>
> radar data (10 parameters taken at approximately 12-second increments)

The CTAS horizontal and vertical data were used to define the CTAS profile. The radar data were not used in this task.

GPS files contained the same horizontal and vertical CTAS trajectory data and GPS-derived data (10 parameters) taken at approximately 1-second increments.

The abbreviated and complete FMS files contained airborne-derived data (30 parameters for the abbreviated set and 84 parameters for the complete set) taken at 1-second intervals. The radar, GPS and FMS data were in the form of time series. The CTAS data were in the form of discrete horizontal and vertical data points. The vertical data had time records; the horizontal data did not have time records.

In order to make maximum use of all of the available data, it was determined that the data needed to be in consistent formats. Since the GPS and FMS data were in time series format, a methodology was developed to convert the CTAS discrete-event data to time series. To accomplish this, a method was developed of integrating the vertical trajectory data to produce a time series of CTAS parameters. The resulting horizontal values were compared to the known discrete CTAS horizontal data points. One problem encountered was the resolution of the CTAS data (ground speed data was 1 knot and true course was 0.1 degree). With integration times of 500 to 700 seconds, small errors accumulated in the resulting X, Y positions. To compensate for this source of error, an iterative process was developed that compared the X, Y positions with the known X, Y positions from the CTAS horizontal trajectory data. This technique produced differential corrections that were applied to the integrated position. This process produced CTAS time series that agreed with the X, Y positions from the CTAS horizontal trajectory data with an average of about 20 feet and the integration times agreed with the CTAS vertical trajectory times to within 0.1 second. These CTAS time series replicated the discrete CTAS horizontal and vertical profile data points with sufficient accuracy to allow the objectives of the vertical trajectory assessment to be achieved.

Once the CTAS time series were available, the CTAS, GPS, and abbreviated FMS data sets were time-merged. Specific parameters from the complete FMS data set (spoiler settings) were added to the time-merged data sets. The data set (61 parameters) contains CTAS, GPS and FMS data at 1-second time intervals (the GPS time marks).

Analysis Methodology

The first step in the analysis of vertical profile errors was to account for the descent initiation error. The error caused by early (or late) initiation of descent is determined by analytically shifting the CTAS profile in distance (or time) so that the TOD for the aircraft and the CTAS profile are coincident. This process is shown graphically in Figure 2. The original CTAS profile (shown in green) is shifted in distance so that the TOD of the shifted CTAS profile (shown in red) is coincident with the aircraft profile (shown in blue). The altitude error due to early (or late) initiation of descent is calculated by subtracting the altitude of original CTAS profile from the shifted CTAS profile. The resulting error (shown in magenta) is shown on the lower half of Figure 2 with the error scale on the right side of the graph. The descent initiation error varies with distance during the descent. During the steeper portions of the descent, typically early in the descent, the error is larger in magnitude than during shallower portions of the descent.

With the aircraft and shifted CTAS profiles coincident at TOD, the two profiles can be compared to allow further analytical processing. Specifically, the error sources identified in the previous section are compared. For some parameters the differences are obvious. These parameters include weight experimental error, weight error, performance degradation, thrust above flight idle, and spoiler deployment. For other parameters, the errors must be calculated by comparing the aircraft and shifted CTAS profiles. These parameters include true airspeed errors, wind gradients, along-track errors caused by differences in ground speed, and atmospheric temperature errors.

The methodology chosen to analyze the vertical profile error made use of the equations of motion for an aircraft in flight. These equations are described in their complete form in [4] and in simplified form using small angle approximations in [1].

Equations for Differential Altitude Corrections

Derivation of Equations for Differential Altitude Corrections

Aircraft Equations of Motion are as follows (from [1], page 33, equations A1 and A2):

$$dV_a/dt = g * (T - D)/ W - g * \gamma - dV_w/dt \qquad \text{(Equation 1)}$$

$$dh/dt = V_a * \gamma \qquad \text{(Equation 2)}$$

where V_a = true airspeed
t = time
g = acceleration of gravity (32.2 ft/sec^2)
T = thrust
D = drag
W = aircraft weight
γ = flight path angle
V_w = horizontal component of tailwind

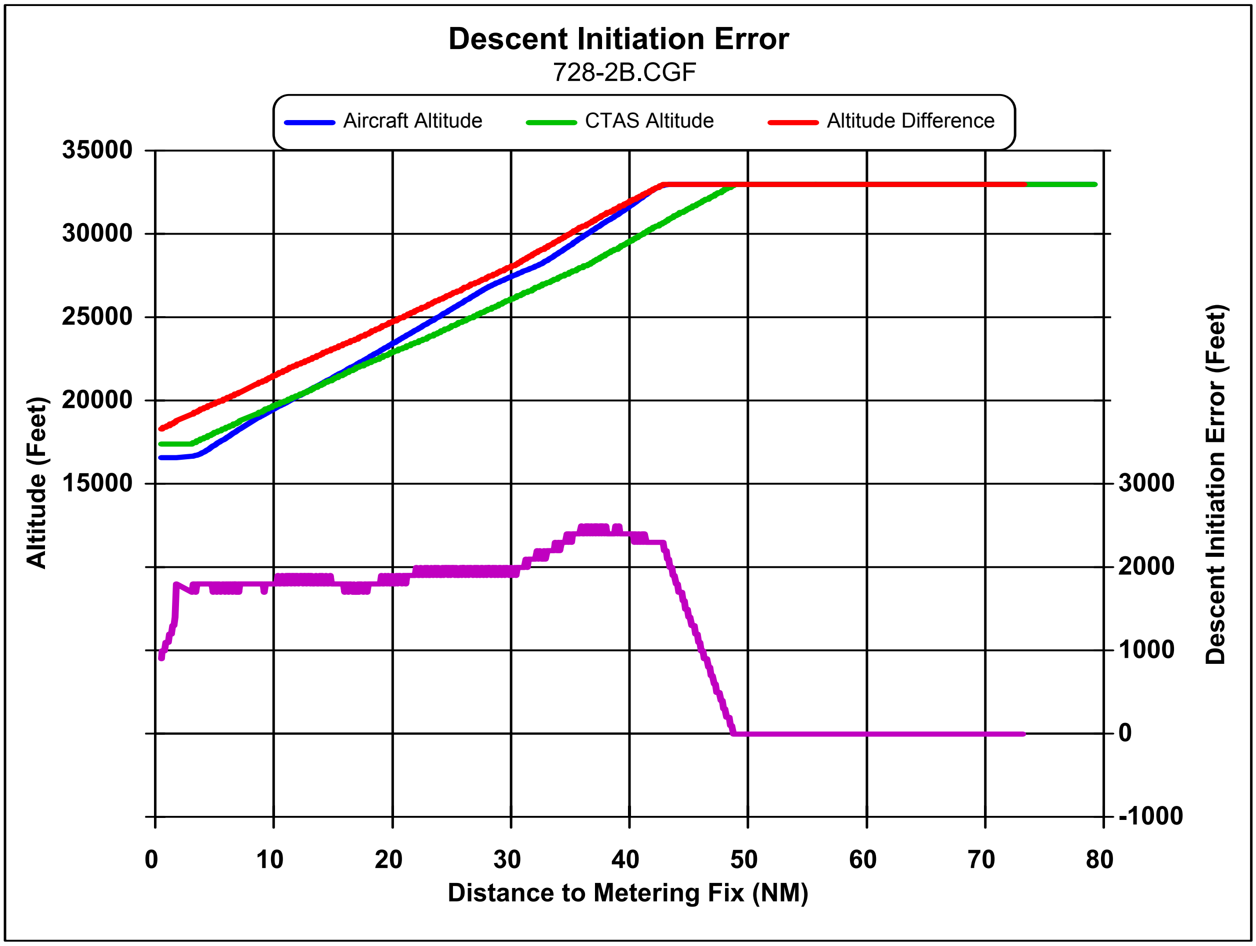


Figure 2 Graphical Representation of Descent Initiation Error for Run 728-2

Substituting (Equation 2) into (Equation 1), Equation 3 is obtained:

$$dV_a/dt = g * (T - D)/W - g * (dh/dt)/ V_a - dV_w/dt \qquad \text{(Equation 3)}$$

Using the chain rule to differentiate the left side of Equation 3, the following is obtained:

$$dV_a/dh * dh/dt = g * (T - D)/W - g * (dh/dt)/ V_a - dV_w/dt$$

Collecting terms and solving for dh/dt, the equation for vertical error evaluation is obtained:

$$dh/dt * (g/ V_a + dV_a/dh) = g * (T - D)/ W - dV_w/dt \qquad \text{(Equation 4)}$$

The advantages of this formulation of the equations of aircraft motion in the vertical dimension are threefold: 1) the flight path angle, γ, is eliminated, 2) it results in a single equation, and 3) it provides a means of handling constant Mach and calibrated airspeed descent schedules through the term dV_a/dh, which is discussed in the next paragraph.

For simplicity, use the following notation

$f(V_a) = 1 / (g/ V_a + dV_a/dh)$

Then solving for dh/dt, the principal equation for vertical error evaluation is obtained:

$dh/dt = f(V_a) * [g * (T – D)/ W – dV_w/dt]$ (Equation 5)

Partial differentiation can now be used to determine the equations for differential altitude corrections as a function of the various error components.

Evaluation of dV_a/dh for Various Mach/CAS Descent Schedules

For constant Mach/CAS descent schedules, the rate of change of airspeed with altitude can be calculated from standard Mach, CAS, TAS formulas found in many aeronautical texts and papers (e.g., Reference 2, page 221, equations 14 and 15).

For the constant Mach segment, dV_a/dh can be represented by a single value at altitudes below approximately 36000 feet, which applies to all descents analyzed herein. For the constant CAS segment, dV_a/dh can be represented by a linear equation of the form:

$dV_a/dh = A + B * (Altitude/1000)$

The result of this analysis (for altitudes of 33000 feet and below) is shown in Table 1.

Table 1 Rate of Change of True Airspeed for Constant Mach/CAS Descent Schedules

Mach/CAS Schedule	Transition Altitude (ISA Conditions)	dV_a/dh for constant Mach	dV_a/dh for constant CAS – A term	dV_a/dh for constant CAS – B term per 1000 ft
0.73/280	29,174	-0.00186/second	0.003294/second	0.000121/second
0.76/240	Above 33,000 feet	N/A	0.002732/second	0.000116/second
0.76/320	24,912	-0.00194/second	0.004001/second	0.000114/second

N/A – Not Applicable

Performance Degradation Model: Sensitivity to errors in the quantity (T – D) (performance degradation of the NASA Boeing 737-100 research aircraft) is determined by differentiating Equation 5 with respect to the quantity "thrust minus drag" (T – D):

$\Delta(dh/dt) = f(V_a) * g / W * \Delta(T – D)$ (Equation 6)

where Δ(T – D) = error in thrust minus drag shown in [1], Appendix A, page 39. As stated in [1], the term Δ(T – D) varies from about 9 percent at FL 330 to about 2 percent at 17,000 MSL. The performance degradation was modeled as follows:

$\Delta(T – D) = (.02 + .07 * (Altitude – 17000) / (33000 – 17000)) * (T – D)$ (Equation 7)

Determination of Thrust Minus Drag Term: For each of the descent cases T – D can be calculated from the CTAS time series. This was done for 6 conventional descent cases and 7 VNAV descent cases. The data show that T – D varies with both the speed of descent and the aircraft weight. This observation can also be assessed from physical arguments as follows:

The general equation for drag is given by the equation:

$$\text{Drag} = \tfrac{1}{2} \rho V_a^2 C_d S \qquad \text{(Equation 8)}$$

where ρ = air density
V_a = true airspeed
C_d = drag coefficient
S = aircraft reference area

During the descent, thrust is small so the drag term dominates. Drag is clearly related to airspeed by the drag equation shown above. The drag coefficient has several components, some of which (induced drag) are a function of lift, which in turn is related to aircraft weight. Other components of drag, such as form drag, are not influenced by aircraft weight. Therefore, it is clear from physical discussions that the term T – D has a component that varies as a function of weight.

The most analytically sound manner to evaluate the quantity T – D is through analysis of the performance of the aircraft thrust and drag characteristics during descent. However, this information was not available for the analysis of vertical profile performance, so an alternative method was employed. This alternative method made use of the available CTAS data to provide initial characteristics of T – D during descent; then adjustments were made to these values to minimize the vertical error residual values.

An initial estimate of T – D values was calculated from the 13 CTAS runs confirmed variations with speed, weight and altitude. The variations in altitude were averaged over the descent to produce 13 values for T – D. These 13 values represent 6 speed schedule/weight combinations (3 speed conditions – low, medium and high, times 2 weight conditions – 85000 and 98000 pounds). These initial estimates demonstrated the expected characteristics of variation with speed and weight:

- the estimates were negative for all 13 descents,
- the magnitude of the estimates became more negative as the speed during descent increased,
- the magnitude of the estimates became more negative as the weight of the aircraft increased.

With only a few data points available, a simple model of characterizing T – D was utilized. The characteristics of the 13 data points suggested that linear equation of the following form was appropriate for the analysis:

$$T - D = A + B * (\text{Weight}/85000) \qquad \text{(Equation 9)}$$

where A and B are constants to be determined,
Weight = weight of the aircraft during descent.

The 13 data points suggested that the constant A was approximately 50 percent of the value of T – D. The value of the constant B was selected to minimize the residual error of the 13 descent cases used in the analysis. A separate evaluation of A and B was used for each of the three descent speed schedules. The resulting values for the constants A and B and the values for T – D are shown in Table 2.

Table 2. Thrust Minus Drag Values Used in the Analysis

Mach/CAS Schedule	Constants		Thrust Minus Drag Aircraft Weight	
	A	B	85,000 pounds	98,000 Pounds
.76/240 KIAS	-3250	-3019	-6269 pounds	-6731 pounds
.73/280 KIAS	-3700	-3437	-7137 pounds	-7663 pounds
.76/320 KIAS	-4100	-3809	-7909 pounds	-8491 pounds

<u>Weight Error and Weight Experimental Error Model:</u> Sensitivity to weight error, ΔW, requires consideration of the functional relationship described by Equation 9. The terms f(Va) and g are independent of weight and therefore are constant with respect to the differentiation process. However the functional relationship of T – D as a function of weight must be considered.

With the formulation for T – D in Equation 9, the variation of altitude rate with weight error can be evaluated as the derivative of Equation 5 with respect to the weight term W. This yields:

$$\Delta(dh/dt) = -f(V_a) * g * A / W^2 * \Delta W \qquad \text{(Equation 10)}$$

where A is the value shown in Table 2
W is the aircraft weight
ΔW is the weight error at TOD (and the weight experimental error value for descents where this error applies).

<u>Wind Gradient Error Model:</u> Sensitivity to wind gradient, $\Delta dV_w/dt$, is determined by differentiating Equation 5 with respect to this term. This yields

$$\Delta(dh/dt) = -f(V_a) * \Delta dV_w/dt \qquad \text{(Equation 11)}$$

The wind gradient term was calculated from a comparison of winds recorded in the aircraft with winds used in the CTAS model. Initially, the wind gradient was calculated by comparing the ground speed minus airspeed values recorded in the aircraft with the ground speed minus airspeed values used by CTAS. This formulation, although easy to calculate, is not correct for the descent speed schedules used in these tests. It erroneously includes aircraft speed changes due to the specific Mach/CAS schedule.

To eliminate the aircraft speed changes, and alternative formulation was used. This formulation uses the wind speed and direction to establish wind vectors for the aircraft and CTAS. The gradients of the two wind vectors are determined by numerically differentiating the wind vector with respect to time. The along-track wind gradient is then determined by taking the dot product of the aircraft (or CTAS) wind gradient and the aircraft (or CTAS) track angle. The difference in the along-track wind gradient is then determined by subtracting the CTAS along-track wind gradient from the aircraft along-track wind gradient.

<u>Thrust Above Flight Idle Model:</u> Sensitivity to additional thrust, ΔT, is found by differentiating Equation 5 with respect to the thrust term T. This yields

$$\Delta(dh/dt) = f(V_a) * g / W * \Delta T \qquad \text{(Equation 12)}$$

Information was obtained from Mr. Dave Williams of NASA Langley on modeling thrust increments above those of flight-idle. This information is as follows:

> "... use throttle position to determine if a correction is needed, and then use the EPR to adjust the thrust. ... use the EPR value prior to the throttle increase as the reference, and then apply a thrust increment based on the following:
>
> | Altitude | Thrust per engine per .1 EPR |
> |---|---|
> | 30000 | 350 pounds |
> | 25000 | 430 pounds |
> | 20000 | 540 pounds |
>
> These numbers seem reasonable across the speed range of the descents. You can either do each engine separately or average the EPRs and double the resulting thrust."

The thrust equation developed from these values is of the form:

$$\text{Thrust} = \{1280 - 49 * (\text{Altitude} / 1000) + 0.6 * (\text{Altitude} / 1000)^2\} * (\Delta\ \text{EPR1} + \Delta\ \text{EPR2}) / 0.1$$

where Δ EPR1 and Δ EPR2 are the change in engine pressure ratio for each engine above that of flight idle at a time just prior to application of thrust.

Spoiler Deployment Model: Sensitivity to additional drag due to deploying the spoiler, ΔD, is given by differentiating Equation 5 with respect to the drag term, D. It yields

$$\Delta(dh/dt) = - f(V_a) * g / W * \Delta D \qquad \text{(Equation 13)}$$

Drag is modeled as $\frac{1}{2} \rho V_a^2 C_d S$. The value of the term $\frac{1}{2} \rho_o C_d S$ was estimated empirically as $0.0495 * \sin \delta_s$, where δ_s is the spoiler deflection angle. The evaluation of this coefficient was found by minimizing the residual vertical error for those descents that had significant use of spoilers.

Airspeed Error Model: Sensitivity to airspeed errors is found by finding the partial derivative of Equation 5. The right side of the equation is independent of airspeed; therefore, its derivative is zero. Differentiating by parts, the following is obtained,

$\Delta(dh/dt) * (g/ V_a + dV_a/dh) - (dh/dt) * g / V_a^2 * \Delta V_a = 0$, or collecting terms

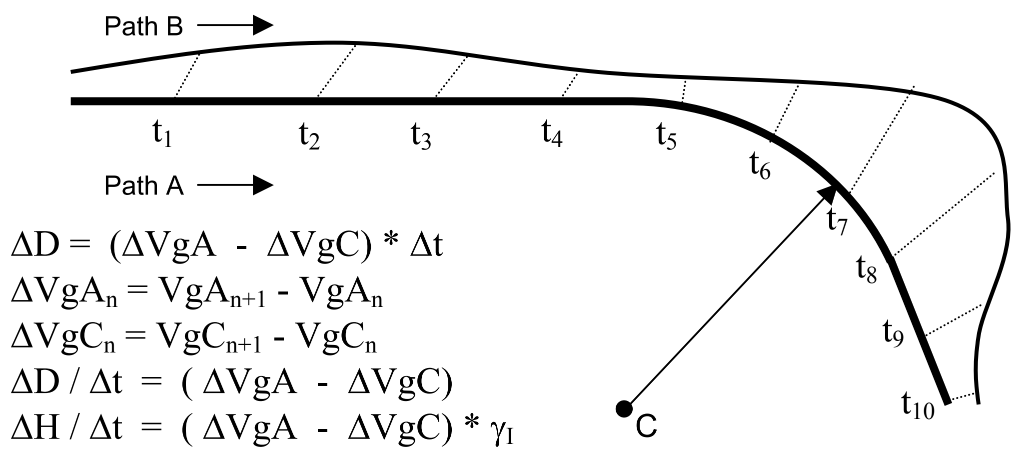
$$\Delta(dh/dt) = (dh/dt) * f(V_a) * g / V_a^2 * \Delta V_a \qquad \text{(Equation 14)}$$

Temperature Error Model: Sensitivity to non-standard atmospheric temperatures is found by finding the partial derivative of equation A4 from Reference 1. That becomes

$$\Delta(dh/dt) = dh_p/dt / T_{k,s} * \Delta T_k \qquad \text{(Equation 15)}$$

where dh_p/dt = rate of change of pressure altitude
$T_{k,s}$ = standard day atmospheric temperature in degrees Kelvin
ΔT_k = error in atmospheric temperature in degrees Kelvin

Along-Track Error Model: The model for evaluating the vertical error due to differences in the along-track position of the aircraft and the CTAS model was determined by the path construction shown in Figure 3. Path A represents the CTAS model of the aircraft path. Path B represents the path of the aircraft. The nominal flight path consists of a straight segment followed by a curved segment centered at point C followed by another straight segment. The time marks, t_1, t_2, ... t_n, represent times of data points along each of the paths. The distance difference, ΔD, between the two paths is shown by the equations in Figure 3. The terms ΔVgA_n and ΔVgC_n are the increment values for ground speed changes for the aircraft and the CTAS model respectively. These incremental changes are determined by subtracting adjacent values of ground speed as shown in the second and third equations in Figure 3. The rate of change of this difference is shown in the 4th equation and is determined by dividing by the Δt term. From this term the impact on the vertical profile is calculated by multiplying by the inertial flight path angle, γ_I.



$$\Delta D = (\Delta VgA - \Delta VgC) * \Delta t$$
$$\Delta VgA_n = VgA_{n+1} - VgA_n$$
$$\Delta VgC_n = VgC_{n+1} - VgC_n$$
$$\Delta D / \Delta t = (\Delta VgA - \Delta VgC)$$
$$\Delta H / \Delta t = (\Delta VgA - \Delta VgC) * \gamma_I$$

Figure 3 Along-Track Error Construction

Along-Track Error – Distance and Along-Track Error – Speed: The along-track error described above can be subdivided into a component due entirely to path adherence, "Along-Track Error – Distance," and a component due to variations in speed, "Along-Track Error – Speed." The path adherence component is determined by projecting the along-track segments of Path B onto Path A and comparing the accumulated distance traveled. "Along-Track Error – Distance" is most prominent at turns where the aircraft may overshoot the turn. Paths flown under control of a FMS, such as the VNAV descents, usually have considerable less "Along-Track Error – Distance" than do aircraft controlled by conventional means of navigation. "Along-Track Error – Distance" is calculated by the projection method identified above. "Along-Track Error – Speed" is determined by subtracting "Along-Track Error – Distance" from the total Along-Track Error identified in the previous paragraph.

Findings - Vertical Descent Performance Errors

Conventional and VNAV Descents

During the 1994 flight demonstrations, there were 6 conventional descents and 19 VNAV descents. Of the 19 VNAV descents, 7 used the FMS predicted profile to identify TOD, 6 used the CTAS TOD, and 6 used the RAA method to identify TOD. In the cases where CTAS and the range-altitude arc were used, the aircraft flew the initial part of the descent using conventional descent procedures. At some point in this conventional descent, the aircraft would intercept the VNAV descent. From that point, the aircraft would follow the VNAV descent. Because CTAS TOD and the RAA TOD cases are mixed conventional/VNAV descents, it was determined that these cases could not be analyzed as being representative of either conventional descents or VNAV descents. Therefore, the analysis was limited to the 6 conventional descents with CTAS TOD and 7 VNAV descents with FMS TOD. The descents are identified in Table 3.

Table 3 Conventional and VNAV Descent Cases Used in the Analysis

Conventional Descents with CTAS TOD

Run No.	Mach No.	CAS (knots)	CTAS Wt. (pounds)	A/C Wt. (pounds)	Wt. Error (pounds)	Wt. Exp. Error (pounds)
729-2	0.76	240	85000	89877	4877	0
730-2	0.76	240	98000	89398	4398	13000
729-4	0.73	280	98000	83362	-1638	13000
733-3	0.73	280	98000	85364	364	13000
729-3	0.76	320	85000	86245	1245	0
729B-5	0.76	320	98000	78586	-6414	13000

VNAV Descents with FMS TOD

Run No.	Mach No.	CAS (knots)	CTAS Wt. (pounds)	A/C Wt. (pounds)	Wt. Error (pounds)	Wt. Exp. Error (pounds)
729B-1	0.76	240	85000	90084	5084	0
732-3	0.76	240	98000	84900	-100	13000
733-4	0.76	240	98000	82373	-2627	13000
728-2	0.73	280	98000	86839	1839	13000
731-2	0.73	280	85000	88743	3743	0
730-1	0.76	320	85000	92831	7831	0
732-1	0.76	320	98000	93141	8141	13000

In the conventional descent cases, the CTAS solution for TOD was used as the TOD point for the aircraft descent. CTAS identified this point as a DME distance from the Denver VORTAC and this distance was provided to the flight crew as the TOD. In the VNAV descent cases, TOD was determined by the FMS. The FMS had the capability of determining the TOD by specifying BOD conditions at the metering fix.

The characteristics of descent initiation errors were depicted in Figure 2. The magnitude of the error increases from zero at the point where either the aircraft or CTAS TOD occurs. If CTAS TOD occurs first, the error grows in a positive direction as shown in Figure 2. If the aircraft TOD occurs first, the error grows in a negative direction. The error grows linearly until TOD occurs for the aircraft (if CTAS TOD occurs first) or CTAS (if aircraft TOD occurs first). The descent initiation error then levels and approaches its peak value (positive or negative) during the early portion of the descent. The peak value occurs at the point where the inertial flight path angle is at its peak negative value. Mach-limited descent schedules typically have this peak near the beginning of the descent when the aircraft speed in at its maximum value during descent. CAS-limited descent schedules typically have a less pronounced peak value because the flight path angle is nearly constant for these descents. Note that because the descent rate is directly related to the inertial flight path angle, changes in the winds may affect the point where the inertial flight path angle is at its peak negative value. After the peak descent initiation error is reached during the Mach-limited portion of the descent, the error declines slowly in magnitude during the CAS-limited segment of the descent. Finally when either the aircraft or CTAS reach BOD, the error declines linearly to zero. The zero point occurs when both the aircraft and CTAS reach BOD. The peak and average descent initiation error for 6 conventional and 7 VNAV descents are shown in Table 4.

Table 4 Altitude Errors Due To Early (Late) Descent Initiation

Descent Type and Run No.	Mach/ CAS Speed Schedule	Time Shift to Line Up CTAS TOD with Aircraft TOD (seconds)	Distance Flown during CTAS Time Shift (NM)	Peak (+ or -) Altitude Error during Time Shift (Feet)	Average Altitude Error during Time Shift (Feet)
Conventional					
729-2	.76/240 KIAS	18	2.578	580	519
730-2	.76/240 KIAS	17	2.354	500	445
729-4	.73/280 KIAS	-1	-0.128	-60	-37
733-3	.73/280 KIAS	8	0.895	400	314
729-3	.76/320 KIAS	-6	-0.766	-440	-354
729B-5	.76/320 KIAS	5	0.690	360	258
VNAV					
729B-1	.76/240 KIAS	-14	-1.863	-440	-401
732-3	.76/240 KIAS	29	3.705	840	771
733-4	.76/240 KIAS	17	1.914	520	481
728-2	.73/280 KIAS	51	6.156	2460	1859
731-2	.73/280 KIAS	27	3.350	1340	1004
730-1	.76/320 KIAS	1	0.133	80	54
732-1	.76/320 KIAS	46	5.869	2760	2012

Note: Positive time shift means that aircraft descends at a later time than CTAS.
(Aircraft Profile is above CTAS Profile)

In four of the conventional descents, the CTAS TOD was within 8 seconds (about 0.9 Nautical Miles (NM)) of the actual aircraft TOD. In two instances the aircraft descended 17 and 18 seconds (about 2.5 NM) after the CTAS TOD. The early (or late) initiation of descent is noted in Table 4 in both time and distance measurements. These time and distance errors produce the peak and average altitude errors shown in the 5th and 6th columns. Sensitivity coefficients that allow estimation of vertical errors from time and distance errors are discussed in a later section of the report.

Because the VNAV TOD procedures are independent of CTAS, these descent initiation errors showed a greater variation than the CTAS TOD procedures. For Run 729B-1, the aircraft descent started before the CTAS descent by 14 seconds. For the remaining six descents, the aircraft descent began after the CTAS descent. These offsets ranged from a small offset of 1 second for run 730-1 to 51 seconds for run 728-2. The 51-second descent initiation error produced vertical errors in excess of 2400 feet at some points during the descent. Similarly, the 46-second descent initiation error produced vertical errors in excess of 2700 feet.

Barometric Correction Error: Initially it was believed that the barometric correction error that occurred in the CTAS descents would cause these descents to be too short in duration because they terminated at an uncorrected barometric altitude that was typically 600 to 700 feet above the uncorrected aircraft barometric altitude. However two error sources, performance degradation and weight experimental error, created situations whereby CTAS descents were sufficiently long to offset the effects of the barometric correction problem and allow the adjusted CTAS altitude (CTAS altitude plus the vertical error components) to reach the same uncorrected barometric altitude as the aircraft.

> Performance Degradation: The TSRV aircraft had increased drag due to the performance degradation error, which caused the actual aircraft to descend faster than the CTAS model.
>
> Weight Experimental Error: Many of the CTAS descents used the higher aircraft weight of 98000 pounds, which caused the CTAS model to descent at a slower rate than the actual aircraft.

Both of these errors tend to cause the CTAS descent to take longer than had these errors been included in the CTAS model. As a result, most of the CTAS descents were “long” enough to so that CTAS altitude with adjustments for the component errors reached the uncorrected barometric altitude of the aircraft. This occurred for all conventional runs and 6 of the 7 VNAV runs. Only for Run 730-1 was the CTAS descent too short to allow the adjusted CTAS altitude to reach the same uncorrected barometric altitude as the aircraft. This run did not have the weight experimental error and the aircraft weight was 7831 pounds above the nominal CTAS weight of 85000 pounds. This caused the actual aircraft descent to take longer than the CTAS descent and the adjusted CTAS descent terminated at an altitude above the uncorrected barometric altitude of the aircraft. Because this event only occurred on this one run, the barometric correction error did not have a major influence on the analysis of the vertical profile errors. For this one run, the descent analysis was terminated based on the CTAS BOD, while for the other 12 runs, the descent analysis was terminated based on the aircraft BOD.

Results – Conventional Descents

Using the aircraft vertical profiles and the CTAS-shifted profiles, the error models described in the methodology section of the report were applied to determine the component error magnitudes for each of the error sources. The procedure used in determining these error components was to use repeated iterations of the model and assessments of the residual errors to refine the elements of the model with the goal of achieving residual errors with a magnitude of 200 feet or less. For the 6 conventional descents, this goal was achieved on 3 of the 6 runs. One run (729-2) had a residual of 304 feet, which was within 104 feet of the goal. Run 729-4 was within 37 feet of the goal and Run 733-3 was within 8 feet or the goal. It is believed that with additional iterations and refinement of the “thrust minus drag” model, residuals that were within the goal of 200 feet could be produced.

The results of the vertical profile analysis for the 6 conventional descents are presented in tabular form in Table 5. These results are summarized in graphical format in Figure 4, which presents the maximum, minimum, and average error for each component error. The order in which the error components are shown in Figure 4 was selected to allow the legend to be placed in the lower right corner of the graph. The measurement point for the error components in Table 5 and Figure 4 is at a point 250 feet above aircraft BOD or CTAS BOD, whichever occurs first. The

Table 5 Results of Vertical Profile Analysis – Conventional Descents (all values in feet)

Descent Number	729-2	730-2	729-4	733-3	729-3	729B-5
Speed Schedule	.76/240	.76/240	.73/280	.73/280	.76/320	.76/320
Aircraft Minus CTAS(Shifted)	**-3535**	**-1409**	**-2050**	**-1085**	**-400**	**-1181**
Airspeed Error	93	270	159	287	27	252
Performance Degradation	-1083	-1226	-1086	-1068	-1006	-1008
Weight Experimental Error	0	-1349	-1186	-1214	0	-1145
Weight Error	510	456	-149	34	137	-565
Wind Gradient	735	175	779	204	995	1368
ATE Distance	-628	-41	-710	24	-636	-794
ATE Speed	305	567	474	-72	231	541
Thrust Above Idle	0	14	0	1221	105	400
Spoiler Deployment	-3654	-43	-33	-196	-393	-66
Temperature	-117	-44	-60	-99	90	0
Sum of Error Components	**-3839**	**-1221**	**-1813**	**-877**	**-449**	**-1018**
Residual Error	**304**	**-188**	**-237**	**-208**	**49**	**-163**

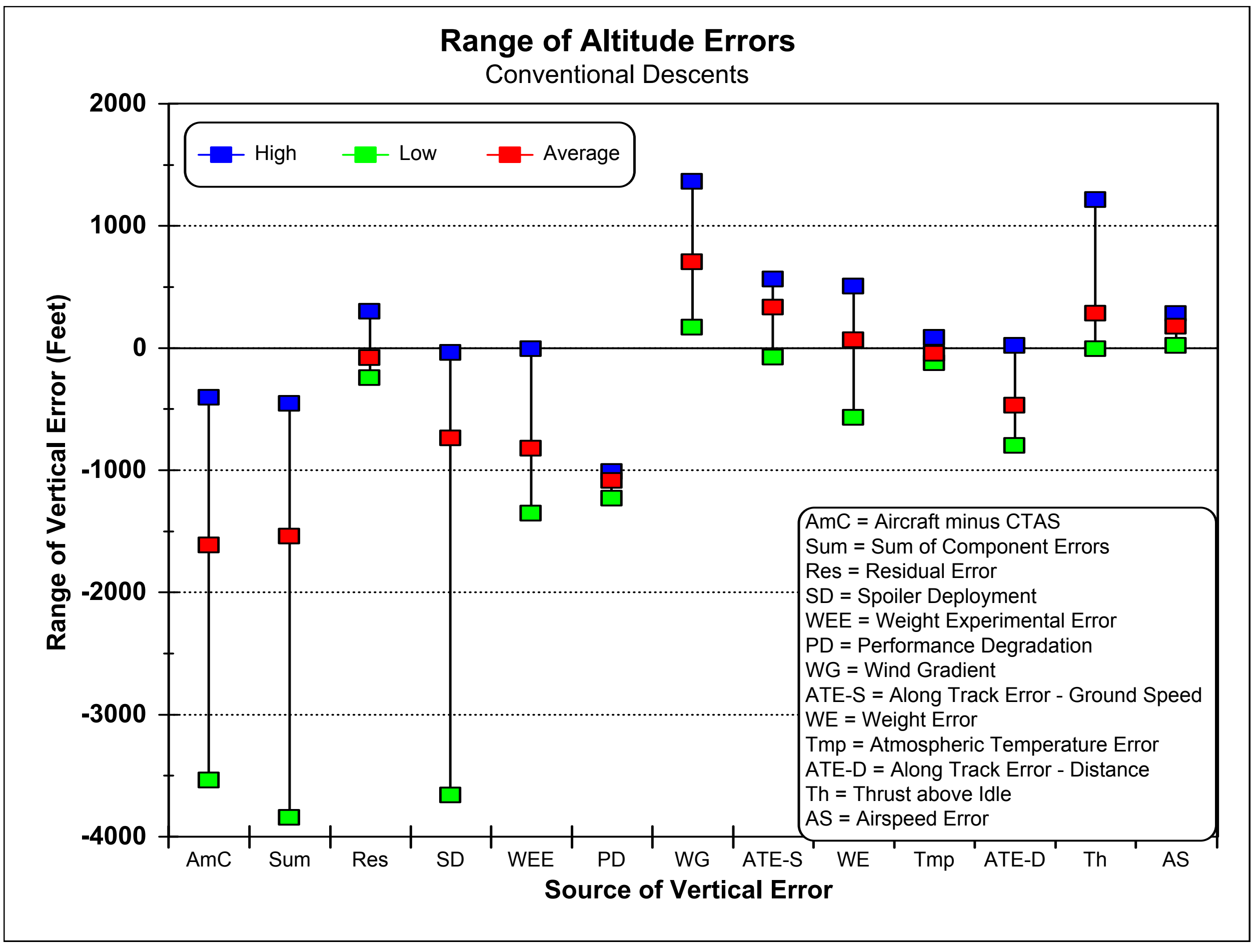


Figure 4 Maximum, Minimum, and Average Values for Vertical Profile Component Errors – Conventional Descents

reason for selecting this measurement point is related to the transition from descending flight to level conditions at BOD. As the aircraft approached BOD, the aircraft configuration was changed from descent to level off at 17000 feet MSL. This configuration change included thrust and control surface changes to reduce the rate of descent. The CTAS model allows instantaneous changes in rate of descent to occur, but this does not happen with the actual aircraft. To avoid having to deal with configuration changes at BOD, the descent error processing was terminated at an altitude that was 250 feet point above aircraft BOD or CTAS BOD, whichever occurred first.

In Table 5, the first two rows show the descent identification number and the corresponding speed schedule used in the descent. The columns are ordered first by the speed schedule ranked from slow speed to high speed. The columns are then ordered by descent identification number. The third row contains the value of the aircraft altitude minus the shifted CTAS profile altitude. This row is highlighted in Table 5 because these are the target values that the sum of the component errors from the model should achieve. Two other rows are highlighted in the table. These are the Sum of Error Components and Residual Error. These three rows summarize the major elements of the vertical error analysis process.

The next 10 rows in Table 5 contain the component errors calculated from the error models. The range of component errors is also shown in Figure 4. The component error values are discussed in the following paragraphs. Where appropriate, source errors and resulting altitude errors are discussed in this section. A later section of the report discusses the sensitivity values in greater detail. Error sensitivity values permit estimation of altitude errors from source errors.

<u>Aircraft Minus CTAS (Shifted) Error:</u> These errors range from a maximum of –400 feet (Run 729-3) to a minimum of –3535 feet (Run 729-2). The large negative error in Run 729-2 was due primarily to a significant use of the spoilers during the descent. In this descent, the aircraft TOD began 18 seconds after the CTAS TOD. Apparently the late initiation of descent caused the crew to apply spoilers to increase the descent rate to achieve the desired terminal conditions at BOD. In retrospect, the aircraft actually arrived at the terminal conditions well before reaching BOD. This demonstrates the considerable altitude error range that can occur in the open-loop conventional descents.

<u>Airspeed Error:</u> The airspeed errors were typically quite small ranging from a maximum of 287 feet to a low of 27 feet. In general, the flight crews adhered to the desired descent speed schedules in the conventional descents.

<u>Performance Degradation:</u> As expected, the performance degradation errors were closely bunched in a range from -1226 feet to –1006 feet. The slower speed descents had slightly more negative errors than did the higher speed descents.

<u>Weight Experimental Error:</u> The source error was either zero for those cases where the CTAS weight was 85000 pounds or 13000 pounds for those cases where CTAS weight was 98000 pounds. For those cases that had the experimental error, the vertical error ranged from –1349 feet to –1145 feet. As with the performance degradation error, the slower speed descents had slightly more negative errors that did the higher speed descents.

<u>Weight Error:</u> Source errors for weight error ranged from –6414 pounds to 4877 pounds. This resulted in corresponding vertical errors of –565 feet and 510 feet. For Run 729B-5, the combination of weight error and weight experimental error is –19414 pounds. This error is

approximately 20 percent of the weight of 98000 pounds used in the CTAS model. The error models used in the vertical profile analysis rely on the source error values being within the limits of the linear range assumed by the model. A source error of 20 percent may be near the limit of the linear range. Even with this large source error, the residual error for this run is well within the goal of 200 feet.

Wind Gradient: The source error for wind gradient errors is the difference in wind gradient experienced by the aircraft and that used by CTAS. The resulting vertical error ranged from 175 feet to 1368 feet. This error source certainly has a significant impact on the vertical profile. It is also a significant contributor to short term variations (noise) in the profile. The aircraft measurement of wind speed and direction has some high frequency noise and is the source of most of the noise observed in the resulting vertical error.

Along-Track Error – Distance: ATE-Distance is a measure of the adherence to the nominal flight path. ATE-Distance is most prevalent at route turn points. With conventional navigation methods, pilots sometimes overshoot the turn and then correct the aircraft track back to the flight plan track. This causes the aircraft path to be longer than the CTAS path. With all other errors being equal, the longer path of the aircraft causes the aircraft altitude to be below the CTAS profile. As seen in Table 5, the ATE-Distance errors range from a low of –794 feet (Run 729B-5) to 24 feet (Run 733-3). Three other cases (Runs 729-4, 729-3 and 729-2) exhibited ATE-Distance errors of –710 feet, -636 feet, and –628 feet, respectively.

Along-Track Error – Speed: ATE-Speed is the remaining along-track error after ATE-Distance is considered. It is the result of speed differences between the aircraft and CTAS in the along-track direction. The largest part of this error is caused by differences in the along-track wind speed. This may occur due to changes in the wind or changes in the aircraft track. For the conventional cases the resulting vertical error ranged from –72 feet (Run 733-3) to 567 feet (Run 730-2).

Thrust Above Flight Idle: The addition of thrust above flight idle results in the slowing of the aircraft's descent rate. Pilots will add thrust if they believe that the aircraft will arrive at BOD well before reaching the metering fix. Thrust application occurred in four of the six conventional descents. The resulting vertical errors ranged from 14 feet (Run 730-2) to 1221 feet (Run 733-3).

Spoiler Deployment: Spoiler deployment results in an increase in the descent rate. Pilots will deploy spoilers if they believe that the aircraft will arrive at BOD above the prescribed altitude and/or above the prescribed airspeed. Significant spoiler deployment occurred in one case (Run 729-2), which resulted in an altitude error of –3654 feet. Lesser degrees of spoiler deployment caused altitude errors ranging from –393 feet (Run 729-3) to –33 feet (Run 729-4).

Outside Air Temperature Errors: Differences in Outside Air Temperature (OAT) between the aircraft and CTAS caused small errors in the vertical profile. These errors ranged from a low of –117 feet (Run 729-2) to a high of 90 feet (Run 729-3). Temperature errors were not a significant source of vertical error during the flight demonstrations.

Sum of Error Components: This row in Table 5 is the sum of the 10 vertical error components in the proceeding rows. The data in this row is the result of applying the error models. The desired effect is to have this sum equal the Aircraft Minus CTAS (Shifted) value described in an earlier paragraph. Ideally, the two values are within the goal of 200 feet. The range of values for the sum of the component errors ranged from a low of –3839 feet (Run 729-2) to –449 feet (Run 729-3).

Residual Error: This row is the difference between the Aircraft Minus CTAS (Shifted) row and the Sum of Error Components Row. These values are a measure of how well the model transforms the sources of vertical error into the actual vertical errors. The residuals ranged from a low of –237 feet (Run 729-4) to 304 feet (Run 729-2). The goal of achieving a residual error of 200 feet or less was achieved in three of the six conventional descents. It is believed that with some refinement of the Thrust Minus Drag model lower residuals can be achieved.

Results – VNAV Descents

The results of the vertical profile analysis for the 7 VNAV descents are presented in tabular form in Table 6. These results are summarized in graphical format in Figure 5, which presents the maximum, minimum, and average error for each component error. As with the conventional descents, the measurement point for the error components in Table 6 and Figure 5 is at a point 250 feet above aircraft or CTAS BOD, whichever occurs first.

The formats of Table 6 and Figure 5 are the same as that for Table 5 and Figure 4. The rows identified as Aircraft Minus CTAS (Shifted), Sum of Error Components, and Residual Errors are highlighted to identify them as the key elements of the vertical error analysis process.

Aircraft Minus CTAS (Shifted) Error: These errors range from a maximum of –305 feet (Run 730-1) to a minimum of –2638 feet (Run 732-1). The large negative error in Run 732-1 was due primarily to three sources, weight experimental error (-1074 feet), performance degradation (-1037 feet) use of the spoilers (-644 feet), and wind gradient (-511 feet). In the VNAV descents, the flight crew (or the flight control system) strives to keep the aircraft on the specified VNAV descent profile. As a result, it is expected that these descents would exhibit a greater use of thrust and/or spoilers than would the conventional descents.

Airspeed Error: The airspeed errors were typically quite small ranging from a maximum of 358 feet to a low of 41 feet. As observed with the conventional descents, the flight crew adhered to the desired descent speed schedules in the VNAV descents.

Performance Degradation: As observed with the conventional descents, the performance degradation errors were closely bunched in a range from -1189 feet to –1018 feet. As with the conventional descents, the slower speed descents had slightly more negative errors than did the higher speed descents.

Weight Experimental Error: The source error was either zero for those cases where the CTAS weight was 85000 pounds or 13000 pounds for those cases where CTAS weight was 98000 pounds. For those cases that had the experimental error, the vertical error ranged from –1285 feet to –1074 feet. As with the performance degradation error, the slower speed descents had slightly more negative errors that did the higher speed descents.

Weight Error: Source errors for weight error ranged from –2627 pounds to 8141 pounds. This resulted in corresponding vertical errors of –249 feet and 673 feet. For Run 733-4, the combination of weight error and weight experimental error is –15627 pounds. This error is approximately 16 percent of the weight of 98000 pounds used in the CTAS model. As with the conventional run, 729B-5, the 16 percent error approaches the limits of the linear characteristics of the model. Even with this large source error, the residual error for this run (-44 feet) is well within the goal of 200 feet.

Table 6 Results of Vertical Profile Analysis – VNAV Descents

Descent Number	729B-1	732-3	733-4	728-2	731-2	730-1	732-1
Speed Schedule	.76/240	.76/240	.76/240	.73/280	.73/280	.76/320	.76/320
Aircraft Minus CTAS(Shifted)	**-1044**	**-1317**	**-1263**	**-2574**	**-1312**	**-305**	**-2638**
Airspeed Error	63	358	158	155	41	136	58
Performance Degradation	-1189	-1182	-1123	-1018	-1126	-1096	-1037
Weight Experimental Error	0	-1285	-1230	-1076	0	0	-1074
Weight Error	630	-10	-249	152	439	910	673
Wind Gradient	749	-746	350	570	119	531	-511
ATE Distance	-10	10	6	-6	-11	-28	4
ATE Speed	-608	187	-226	135	374	470	-157
Thrust Above Idle	202	1243	1186	0	131	60	190
Spoiler Deployment	-727	-4	0	-1587	-1222	-1046	-644
Temperature	-8	-49	-92	-121	-109	57	-160
Sum of Error Components	**-899**	**-1478**	**-1219**	**-2796**	**-1365**	**-6**	**-2658**
Residual Error	**-145**	**161**	**-44**	**222**	**53**	**-299**	**20**

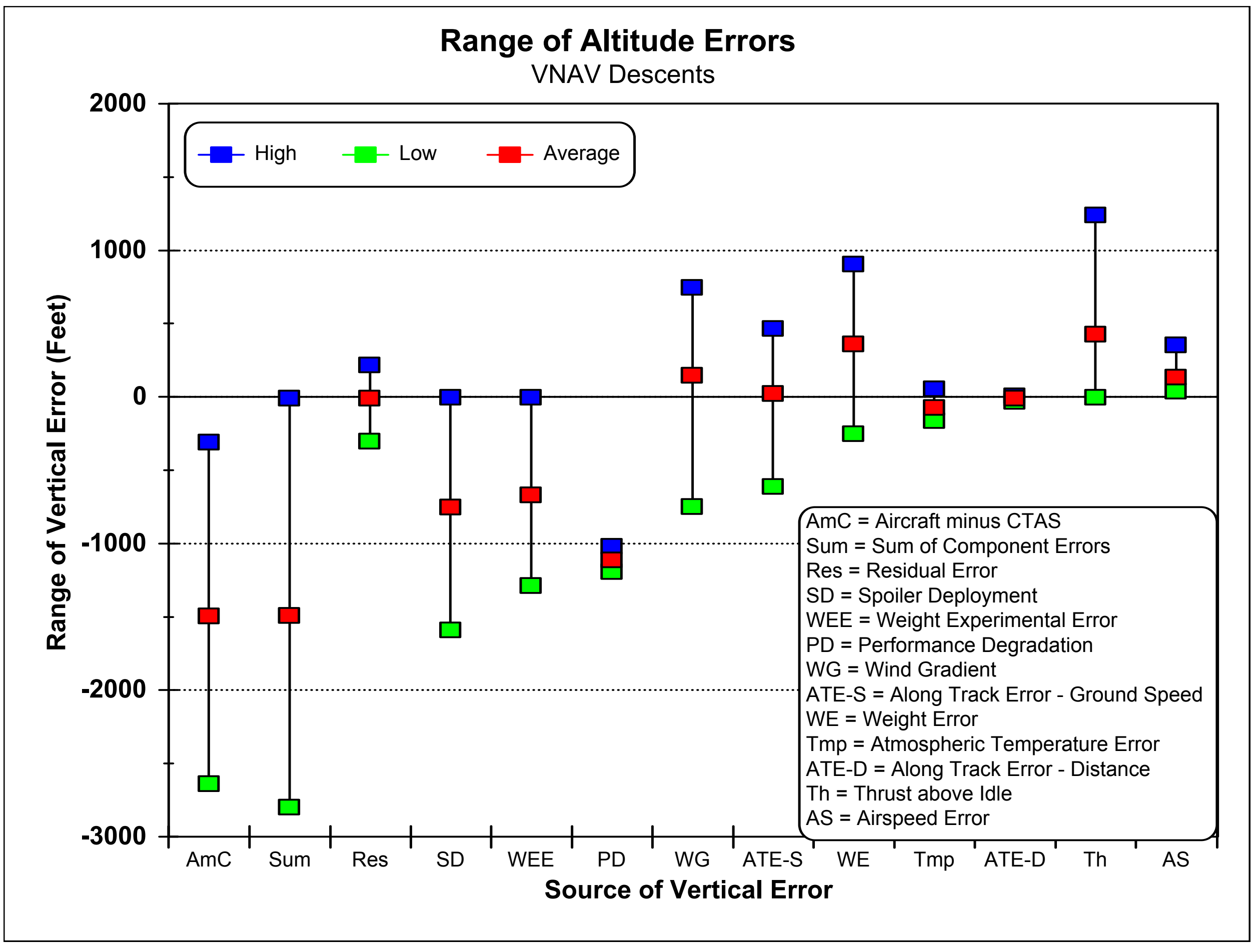


Figure 5 Maximum, Minimum, and Average Values for Vertical Profile Component Errors – VNAV Descents

Wind Gradient: The source error for wind gradient errors is the difference in wind gradient experienced by the aircraft and that used by CTAS. The resulting vertical error ranged from -746 feet to 749 feet. As observed with the conventional descents, this error source has a significant impact on the vertical profile and introduces short-term variations (noise) into the profile.

Along-Track Error – Distance: ATE-Distance is a measure of the adherence to the nominal flight path. For the VNAV descents, the flight crew has positive course guidance throughout the descent including the turn. This guidance should produce a much smaller ATE-Distance error than that observed with the conventional descents. The data supports this observation. Vertical errors caused by ATE-Distance errors for the VNAV cases were very small ranging from a minimum of –28 feet to a maximum of 10 feet.

Along-Track Error – Speed: ATE-Speed error is caused by speed differences between the aircraft and CTAS in the along-track direction. The largest part of this error is caused by differences in the along-track wind speed. For the VNAV cases the resulting vertical error ranged from –608 feet (Run 729B-1) to 470 feet (Run 730-1).

Thrust Above Flight Idle: The addition of thrust above flight idle results in the slowing of the aircraft's descent rate. On the VNAV descents, pilots will add thrust if they observe that the aircraft is flying below the prescribed VNAV profile. Thrust application occurred in six of the seven VNAV descents. The resulting vertical errors ranged from 60 feet (Run 730-1) to 1243 feet (Run 732-3).

Spoiler Deployment: Spoiler deployment achieves the opposite effect from the addition of thrust. Pilots will deploy spoilers if they observe that the aircraft is above the prescribed vertical profile. Significant spoiler deployment occurred in five cases. For these five cases, spoiler deployment caused altitude errors ranging from –1587 feet (Run 728-2) to –644 feet (Run 732-1).

Outside Air Temperature Errors: Differences in Outside Air Temperature (OAT) between the aircraft and CTAS caused small errors in the VNAV descents. These errors ranged from a low of –121 feet (Run 728-2) to a high of 57 feet (Run 730-1). Temperature errors were not a significant source of vertical error during the flight demonstrations.

Sum of Error Components: This row in Table 6 is the sum of the 10 vertical error components in the proceeding rows. The data in this row is the result of applying the error models. The desired effect is to have this sum equal the Aircraft Minus CTAS (Shifted) value described in an earlier paragraph. Ideally, the two values are within the goal of 300 feet. The range of values for the sum of the component errors ranged from a low of –2796 feet (Run 728-2) to –6 feet (Run 730-1).

Residual Error: This row is the difference between the Aircraft Minus CTAS (Shifted) row and the Sum of Error Components Row. These values are a measure of how well the model works in transforming the sources of vertical error into the actual vertical errors. The residuals ranged from a low of –299 feet (Run 730-1) to 222 feet (Run 728-2). The goal of achieving a residual error of 200 feet or less was achieved for five of the VNAV runs. As with the conventional descents, it is believed that with some refinement of the Thrust Minus Drag model, lower residuals can be achieved.

Graphical Results of the Vertical Profile Analysis

Graphical results of the vertical profile analysis are presented in the appendixes. Appendix A contains the six conventional descents. Appendix B contains the seven VNAV descents. The descents are arranged by speed schedule, from low speed to high speed. Within the same speed schedules, the descents are ordered with respect to run number.

For each run, four graphs are presented:

1. The first graph depicts the aircraft and CTAS altitudes as a function of distance from the metering fix. These graphs are based on source data from the aircraft and CTAS. The altitude difference (shown on the right-hand scale) is shown on the bottom part of the graph.
2. The second graph depicts the aircraft and shifted CTAS profile. These graphs show the aircraft and shifted CTAS profile as coincident at TOD. Also shown in the bottom part of the graph is the altitude difference between these two profiles.
3. The third graph depicts the aircraft and the adjusted CTAS profile. For the VNAV descents, the VNAV profile predicted by the FMS is also shown. In all instances, the aircraft profile and the VNAV profile are nearly coincident indicating that the flight crew did a good job of keeping the aircraft on the VNAV profile defined by the FMS. The adjusted CTAS profile is the shifted CTAS profile plus the sum of the 10 component errors. Ideally, the adjusted CTAS profile would overlie the aircraft profile (and the VNAV profile, if applicable) throughout the descent. In some instances the profiles are quite close to being coincident. In other cases, there is a significant difference (±1000 feet or more) in the profiles in the middle portions of the descent. The reasons for these differences are discussed in the section that follows. In the bottom half of the graph, the difference between the aircraft profile and the adjusted CTAS profile is shown. The scale for this graph is on the right-hand side of the graph.
4. The fourth graph for each run presents in graphical form the data shown in Tables 5 and 6. These data include the aircraft minus the shifted CTAS profile, the 10 component errors, the sum of the component errors, and the residual error.

Observations Regarding the Vertical Profile Analysis

Thrust Minus Drag Model: Several times in the development of this report it has been stated that the model for thrust minus drag should be reassessed. The model described in Equation 9 works reasonably well, but the residuals for several runs remained beyond the goal of 200 feet. The model contained herein represents a compromise between accounting for the dependence of thrust minus drag on aircraft weight and the need to develop a model from only 13 data points. The most appropriate development of the thrust minus drag model is to use aerodynamic principles to develop this model and then to validate the model using theoretical analysis or test data.

Errors in the Middle Portion of the Descents: In some runs, the graphical results for the aircraft and adjusted CTAS profiles, shown in Appendixes A and B, have significant errors (1000 feet or more in magnitude) during the middle portion of the descent. These errors are more prevalent in the medium and high speed descents. It is believed that the simplified formulation of the Thrust Minus Drag term in the vertical profile model is a significant contributor to the errors in the middle portion of the descent. Another possible source of error in these descents is the transition from Mach-limited to CAS-limited speed control during the descents. If these transitions occur at different times in the aircraft and CTAS descents, differences between the profiles will occur.

Wind Gradient: Wind gradient errors are a significant source of vertical error in many of the descents. A contributor to the vertical profile error in the middle portion of the descent may be some latency in the wind information available from the FMS data. This data is typically smoothed (filtered) by the FMS to reduce the amount of noise in the wind speed and direction. This latency could introduce some amount of error into the wind gradient error model, which would translate into some error in the adjusted CTAS profile in the mid-portion of the descent. It is believed that this error is a minor contributor to the vertical profile error in the mid-portion of the descent.

Residual Errors and Other Metrics

The principal metric used in the vertical profile analysis is the residual error. Two other metrics are presented in this section to assess the magnitude of the residual error with respect to the total magnitude of component errors. To provide some measure of the magnitude of the 10 component errors, two parameters were considered. The first is the root-sum-square (RSS) of the component errors. The second is the sum of the absolute value (SAV) of the 10 component errors. The ratio of the absolute value of the residual error to either the RSS value or the SAV value provides a measure of the percentage of error that remains unaccounted for by the residual error term. These ratios are shown in Table 7. The residual to RSS ratio percentage ranges from a minimum of 1.1 percent (Run 732-1) to a maximum of 15.7 percent (Run 730-1). The residual to SAV ratio percentage ranges from a minimum of 0.5 percent(Run 732-1) to a maximum of 6.9 percent (Run 730-1). Although the RSS and SAV formulations provide different magnitudes of results (the SAV percentage is typically less than half of the RSS percentage), both methods point to Run 730-1 as having the largest error ratio. This analysis shows that the vertical profile model has more difficulty in explaining the errors for this run than it does for most other runs.

Table 7 Residual Errors and Percentage Error Metrics

Conventional Descents							
Descent Name	729-2	730-2	729-4	733-3	729-3	729B-5	
Speed Schedule	.76/240	.76/240	.73/280	.73/280	.76/320	.76/320	
Residual Error	304	-188	-237	-208	49	-163	
RSS of Component Errors	3979	1991	1993	2070	1629	2381	
SAV of Component Errors	7126	4185	4637	4419	3621	6139	
RSS Percentage	7.6%	9.4%	11.9%	10.0%	3.0%	6.9%	
SAV Percentage	4.3%	4.5%	5.1%	4.7%	1.3%	2.7%	
VNAV Descents							
Descent Name	729B-1	732-3	733-4	728-2	731-2	730-1	732-1
Speed Schedule	.76/240	.76/240	.76/240	.73/280	.73/280	.76/320	.76/320
Residual Error	-145	161	-44	222	53	-299	20
RSS of Component Errors	1820	2305	2110	2262	1771	1911	1857
SAV of Component Errors	4185	5074	4620	4820	3571	4334	4509
RSS Percentage	8.0%	7.0%	2.1%	9.8%	3.0%	15.7%	1.1%
SAV Percentage	3.5%	3.2%	0.9%	4.6%	1.5%	6.9%	0.5%

Source Error to Vertical Error Sensitivity Coefficients

As a final effort for this vertical profile analysis, known levels of source errors were introduced into the vertical profile model and the resulting errors assessed. This method of analysis allows ratios of output errors to input errors (also known as transfer functions) to be determined. These values are averages that apply over the valid altitude range of the model (FL 330 to 17000 feet MSL) and the specific aircraft to which the model applies (NASA's Boeing 737-100 TSRV aircraft). The error ratios determined by this methodology are shown in Table 8. These values are useful in making approximate calculations relating sources to vertical errors for the various error components.

Table 8 Altitude Error Sensitivity to Sources of Vertical Error
(Ratio of Vertical Error to Source Error – VE / SE)

	Mach/CAS Speed Schedule			
Error Source	.76/240	.73/280	.76/320	Units
Weight Error	7.74	7.33	6.88	feet/1000 feet of altitude change/1000 pounds of weight error
Performance Degradation	-13.2	-12.2	-11.8	feet/1000 feet of altitude change/percent degradation in performance
Wind Gradient	-1.79	-2.06	-2.33	feet/1000 feet of altitude change/knot of wind change
Airspeed Error	-2.38	-2.25	-2.00	feet/1000 feet of altitude change/knot of true airspeed error
Temperature	-4.00	-4.00	-4.00	feet/1000 feet of altitude change/1 degree C. temperature error
Thrust above Idle	3.77	3.24	2.75	feet/0.1 change in EPR/second of thrust application
Spoiler Deployment	-0.72	-1.06	-1.53	feet/degree of spoiler deployment/second of application
ATE (Distance)	-285	-341	-395	feet/NM of along-path distance error
ATE (Speed)	-40.8	-37.6	-32.6	feet/knot of along-path ground speed error
Descent Initiation (time)	28	38	52	feet/second of delay in descent initiation
Descent Initiation (distance)	213	311	396	feet/NM of delay in descent initiation

Note: Altitude error sensitivity values apply to Boeing 737-100 aircraft descending between the altitudes of 33000 and 17000 feet.

Conclusions and Recommendations

Conclusions

The principal conclusion reached during the performance of this vertical profile analysis is that the linear error model of the vertical profile developed herein explains and quantifies the relationship between error sources and the resulting vertical errors. In this instance the model explained approximately 92 to 96 percent of the errors, leaving residual errors of 304 feet or less. The goal of achieving residuals of 200 feet or less was achieved in 8 of the 13 descents. The goal was not achieved in 5 of the descents. It is believed that the goal of 200 feet could be achieved with refinement of the thrust minus drag model.

Recommendations

The principal recommendation resulting from this investigation is that the quantity "thrust minus drag" deserves additional attention in the modeling process. In particular, the "thrust minus drag" model should be based on the thrust characteristics of the aircraft in descent and the drag characteristics based on aeronautical engineering fundamentals. The limited number of descents available for the analysis work performed in this investigation precluded a detailed evaluation of the "thrust minus drag" model.

References

1. Williams, D., Green, S., “Flight Evaluation of CTAS Trajectory Prediction Process”, NASA TP-1998-208439, Jul. 1998.
2. Green, S.M., Grace, M.P., and Williams, D. H.: " Flight Test Results: CTAS and FMS Cruise/Descent Trajectory Prediction Accuracy," 3rd USA/Europe Air Traffic Management R&D Seminar, Napoli, Italy, June 13-16, 2000
3. Green, S.M., et al., "Field Evaluation of Descent Advisor Trajectory Prediction Accuracy for En-route Clearance Advisories," AIAA-98-4479, AIAA Conference on Guidance, Navigation, and Control, Boston, MA, Aug. 1998.
4. Jackson, M. R., Zhao, Y. J., and Slattery, R. A.; “Sensitivity of Trajectory Prediction in Air Traffic Management,” Journal of Guidance, Control, and Dynamics, Volume 22, No. 2; March-April 1999.

Acronyms

AmC	Aircraft minus CTAS
ARTCC	FAA Air Route Traffic Control Center
AS	Airspeed Error
ATE-D	Along Track Error - Distance
ATE-S	Along Track Error - Ground Speed
BCExpE	Barometric Correction Experimental Error
BOD	Bottom of Descent
BSE	Barometric Setting Error
CAS	Calibrated Airspeed
C_d	Drag Coefficient
CTAS	Center – TRACON Automation System
D	Drag
DME	Distance Measuring Equipment
DST	Decision Support Tool
EDA	En route Descent Advisor
FAA	Federal Aviation Administration
FL	Flight Level
FMS	Flight Management System
g	Acceleration of Gravity (32.2 ft/sec^2)
GPS	Global Positioning System
IAS	Indicated Airspeed
KIAS	Knots Indicated Airspeed
LNAV	Lateral Navigation System
MSL	Mean Sea Level
NASA	National Aeronautics and Space Administration
NM	Nautical Miles
PD	Performance Degradation
RAA	Range – Altitude Arc
Res	Residual Error
RMS	Root-Mean-Square
RSS	Root-Sum-Square
S	Aircraft Reference Area
SAV	Sum of the Absolute Values
SD	Spoiler Deployment
SE	Source Error
Sum	Sum of Component Errors
t	Time
T	Thrust
T – D	Thrust Minus Drag
TAS	True Airspeed
Th	Thrust above Idle
$T_{k,s}$	Standard Day Atmospheric Temperature in Degrees Kelvin
Tmp	Atmospheric Temperature Error

TOD	Top of Descent
TRACON	Terminal Radar Control Facility
TSRV	Transport System Research Vehicle
V_a	True Airspeed
VE	Vertical Error
VNAV	Vertical Navigation System
VOR	Very High Frequency Omni-directional Radio Range
VORTAC	VOR Tactical Air Navigation Facility
V_w	Horizontal Component of Tailwind
W	Aircraft Weight
WE	Weight Error
WEE	Weight Experimental Error
WG	Wind Gradient
γ	Flight Path Angle
ΔD	Distance Difference
ΔT_k	Error in Atmospheric Temperature in Degrees Kelvin
ΔVgA	Incremental Change in Aircraft Ground Speed
ΔVgC	Incremental Change in CTAS Ground Speed
ρ	Air Density

Appendix A

Conventional Descent Graphs

Run: 729-2

Navigation: Conventional Descent with CTAS TOD

Mach/Speed Schedule: .76/240 KIAS

Aircraft Weight: 89,877 pounds

CTAS Weight: 85,000 pounds

Weight Experimental Error: 0 pounds

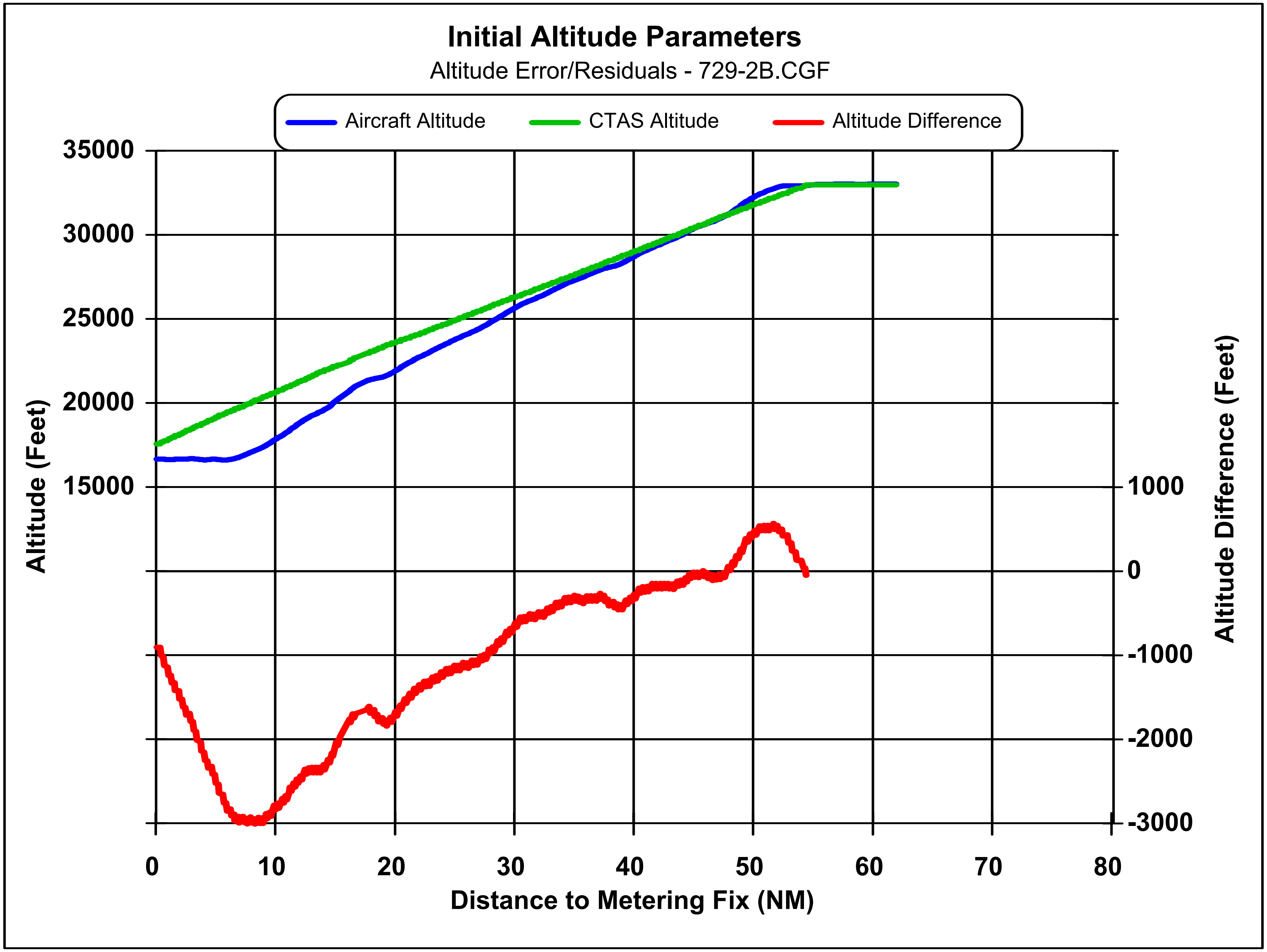
Weight Error: 4,877 pounds

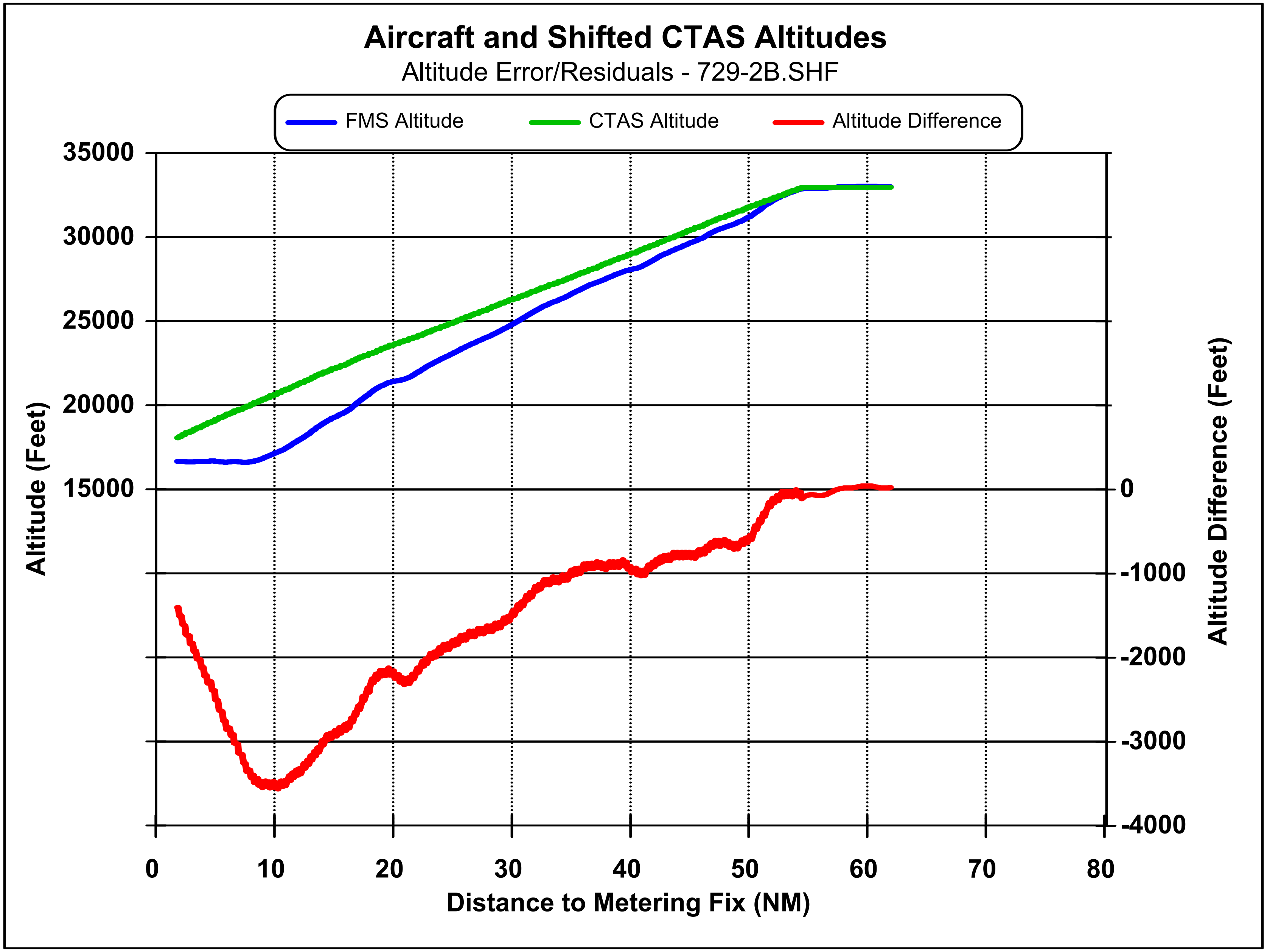
Descent Initiation Error (Time):   18 seconds

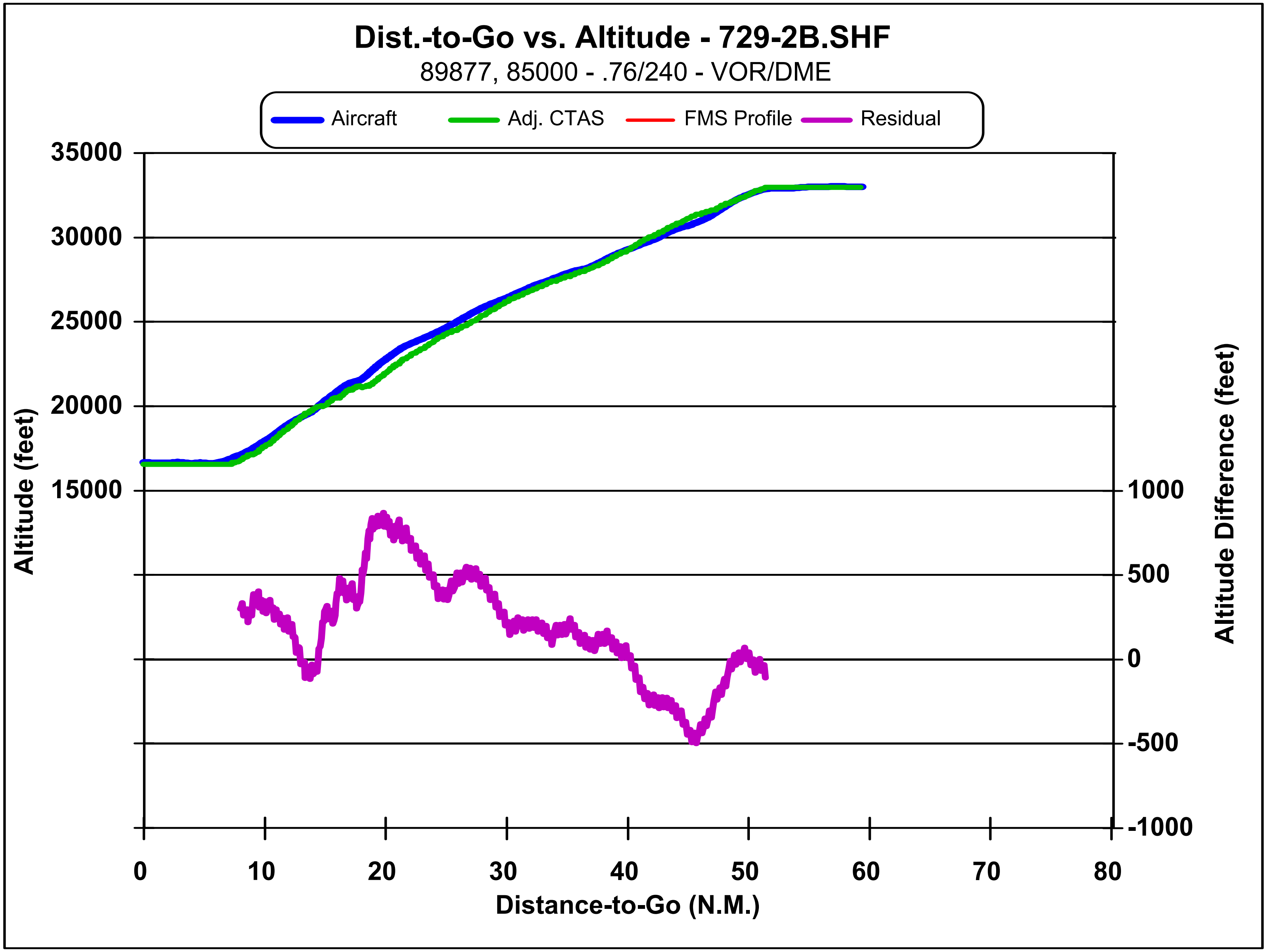
Descent Initiation Error (Distance):   2.578 NM

Vertical Descent Initiation Error (Average): 519 feet

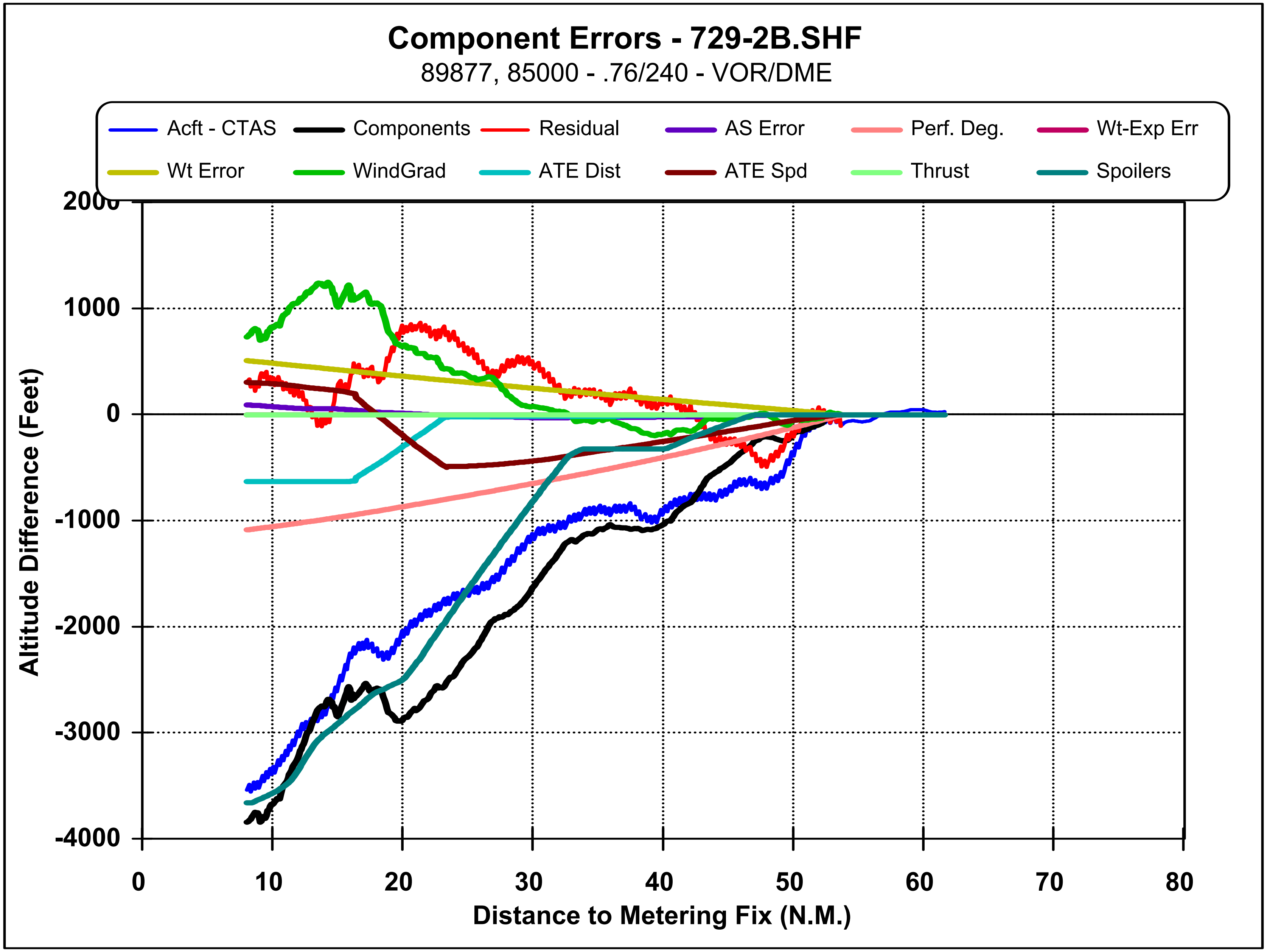
Residual Error: 304 feet


Initial Altitude Parameters
Altitude Error/Residuals - 729-2B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 729-2B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
0
-1000
-2000
-3000
-4000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 729-2B.SHF
89877, 85000 - .76/240 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
Altitude (feet)
35000
30000
25000
20000
15000
Altitude Difference (feet)
1000
500
0
-500
-1000
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)



Component Errors - 729-2B.SHF
89877, 85000 - .76/240 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
2000
1000
0
-1000
-2000
-3000
-4000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 730-2

Navigation: Conventional Descent with CTAS TOD

Mach/Speed Schedule: .76/240 KIAS

Aircraft Weight: 89,398 pounds

CTAS Weight: 98,000 pounds

Weight Experimental Error: 13,000 pounds

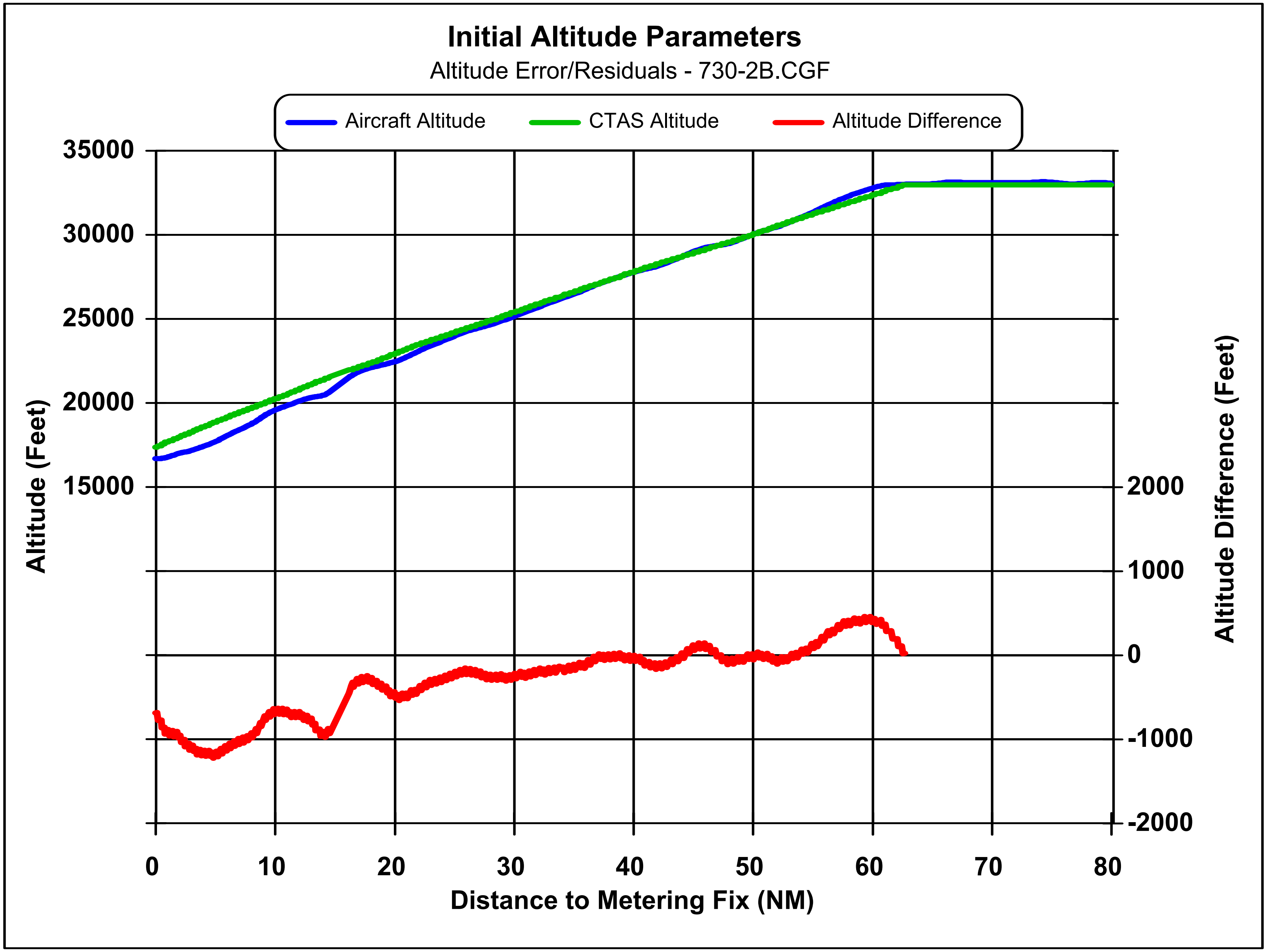
Weight Error: 4,398 pounds

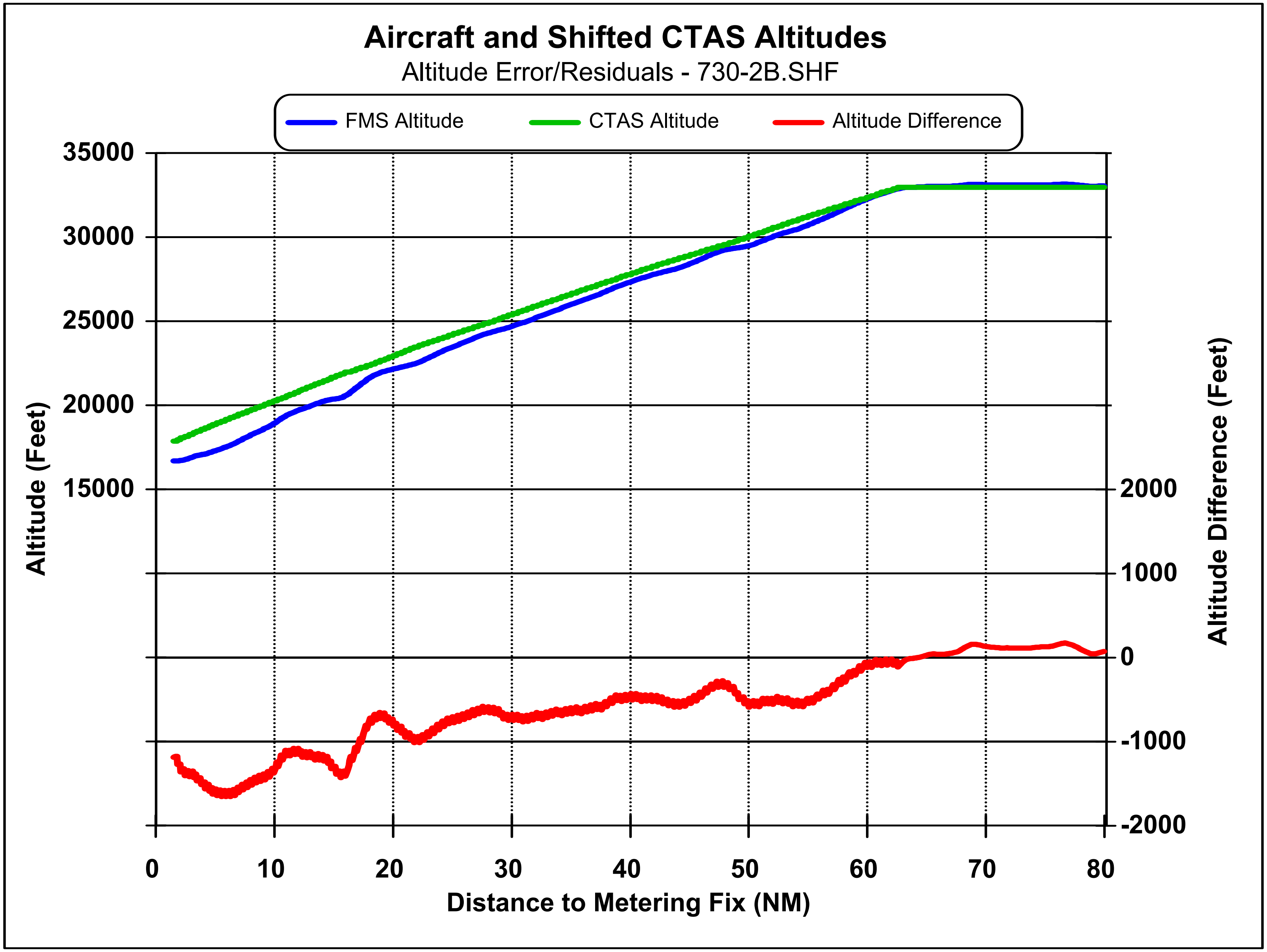
Descent Initiation Error (Time): 17 seconds

Descent Initiation Error (Distance): 2.354 NM

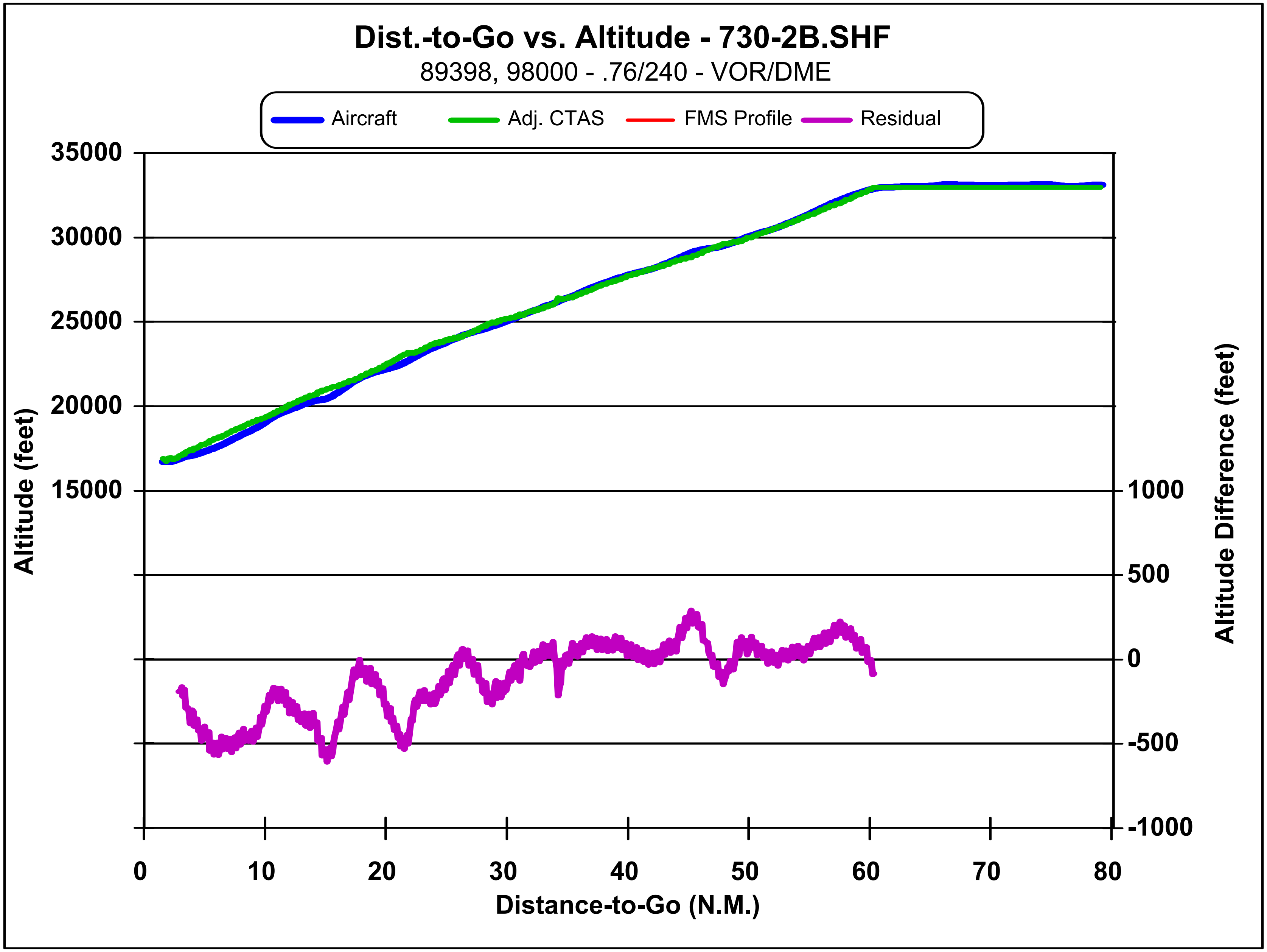
Vertical Descent Initiation Error (Average): 445 feet

Residual Error: -188 feet

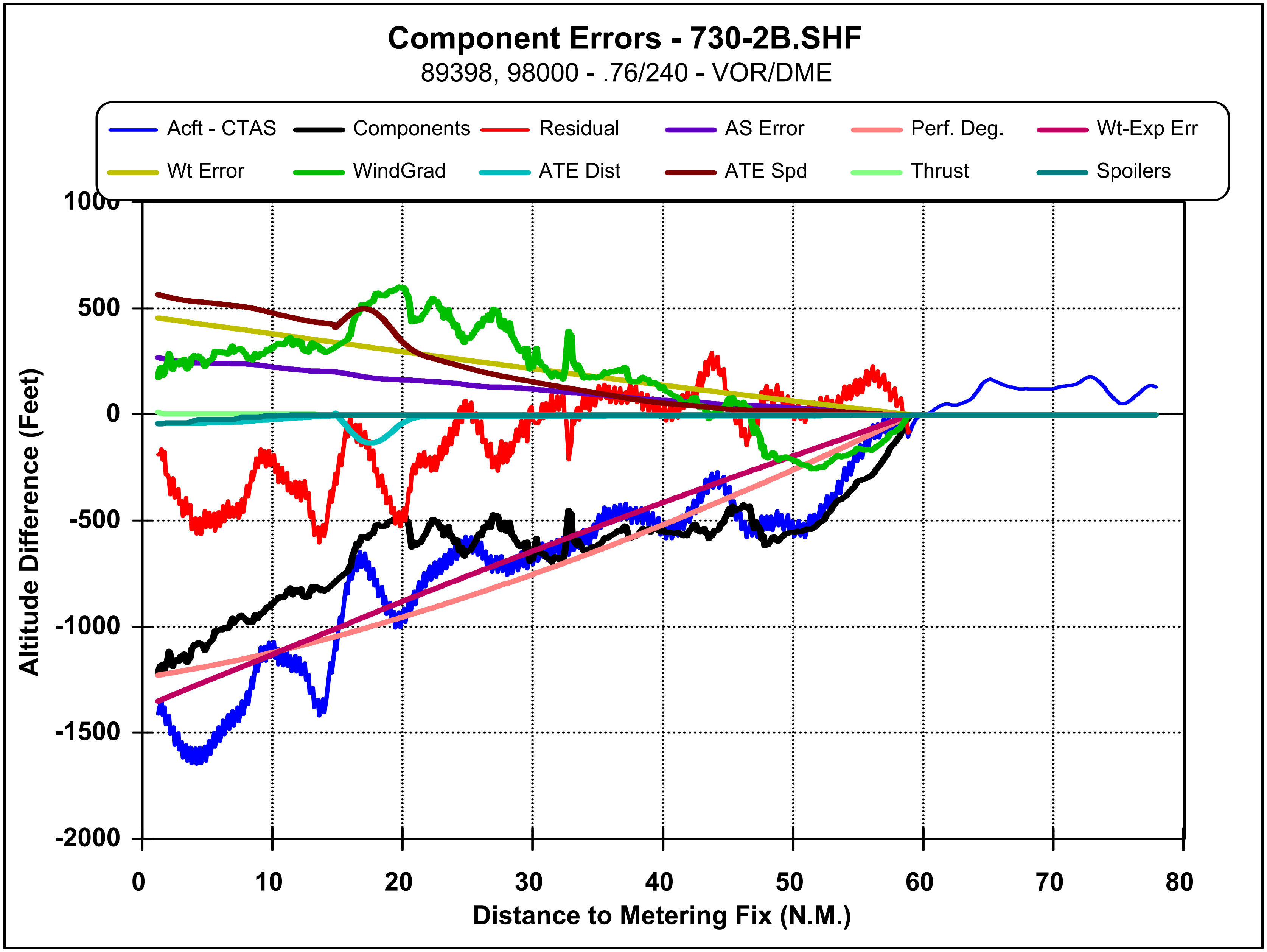

Initial Altitude Parameters
Altitude Error/Residuals - 730-2B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 730-2B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 730-2B.SHF
89398, 98000 - .76/240 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 730-2B.SHF
89398, 98000 - .76/240 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
1000
500
0
-500
-1000
-1500
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 729-4

Navigation: Conventional Descent with CTAS TOD

Mach/Speed Schedule: .73/280 KIAS

Aircraft Weight: 83,362 pounds

CTAS Weight: 98,000 pounds

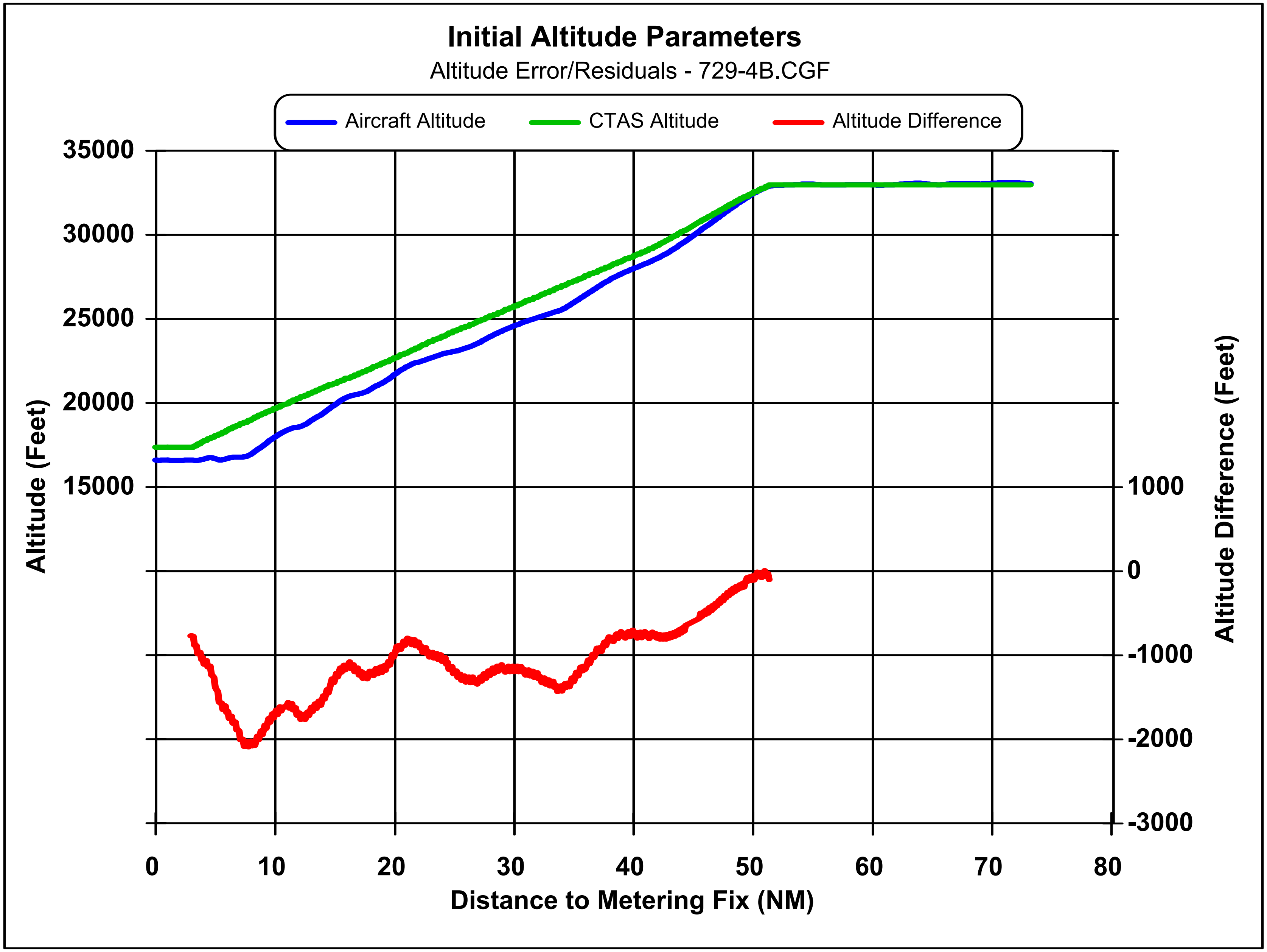
Weight Experimental Error: 13,000 pounds

Weight Error: -1,638 pounds

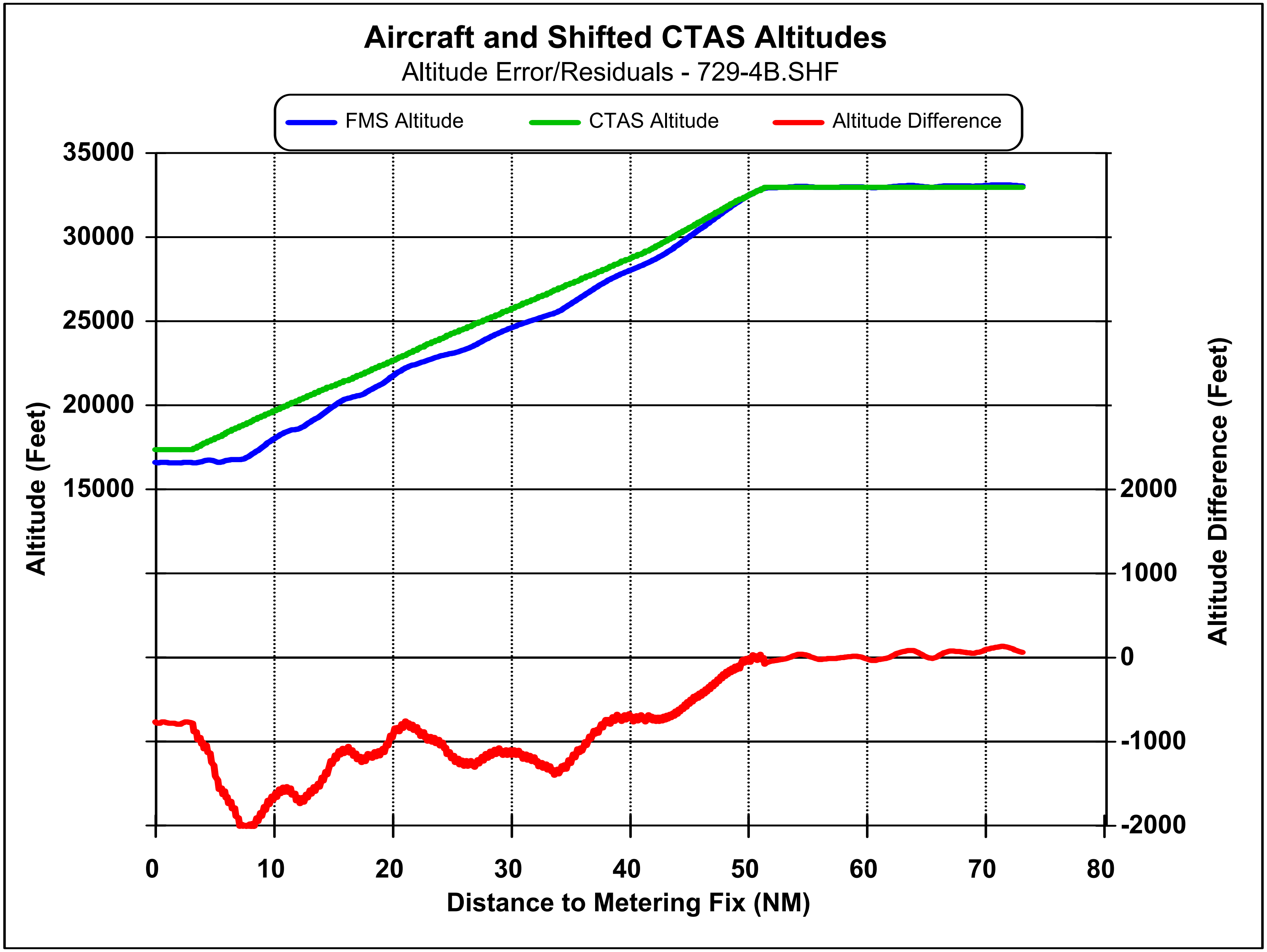
Descent Initiation Error (Time): -1 seconds

Descent Initiation Error (Distance): -0.128 NM

Vertical Descent Initiation Error (Average): -60 feet
Residual Error: -237 feet



Initial Altitude Parameters
Altitude Error/Residuals - 729-4B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 729-4B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
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30000
25000
20000
15000
Altitude Difference (Feet)
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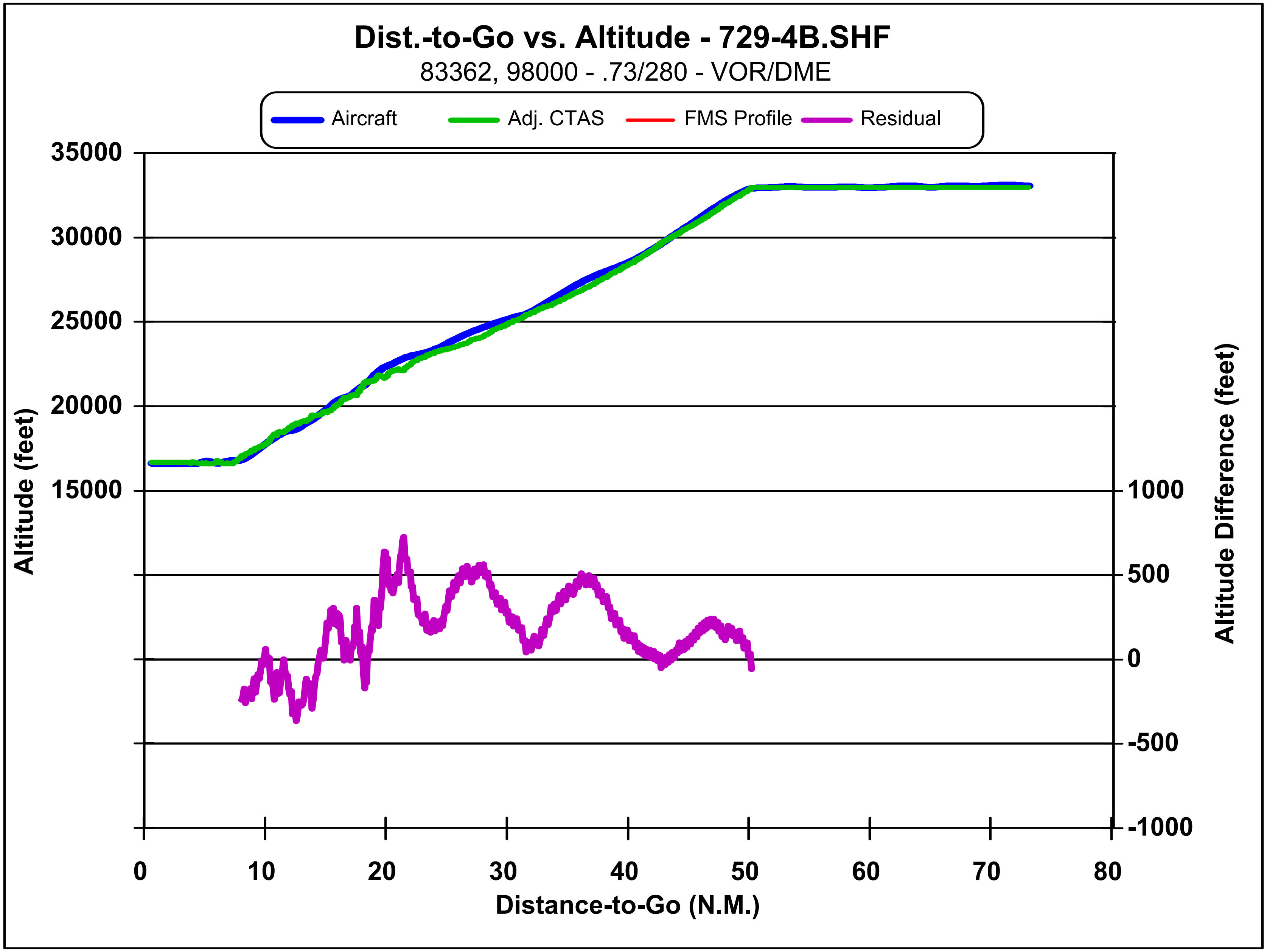
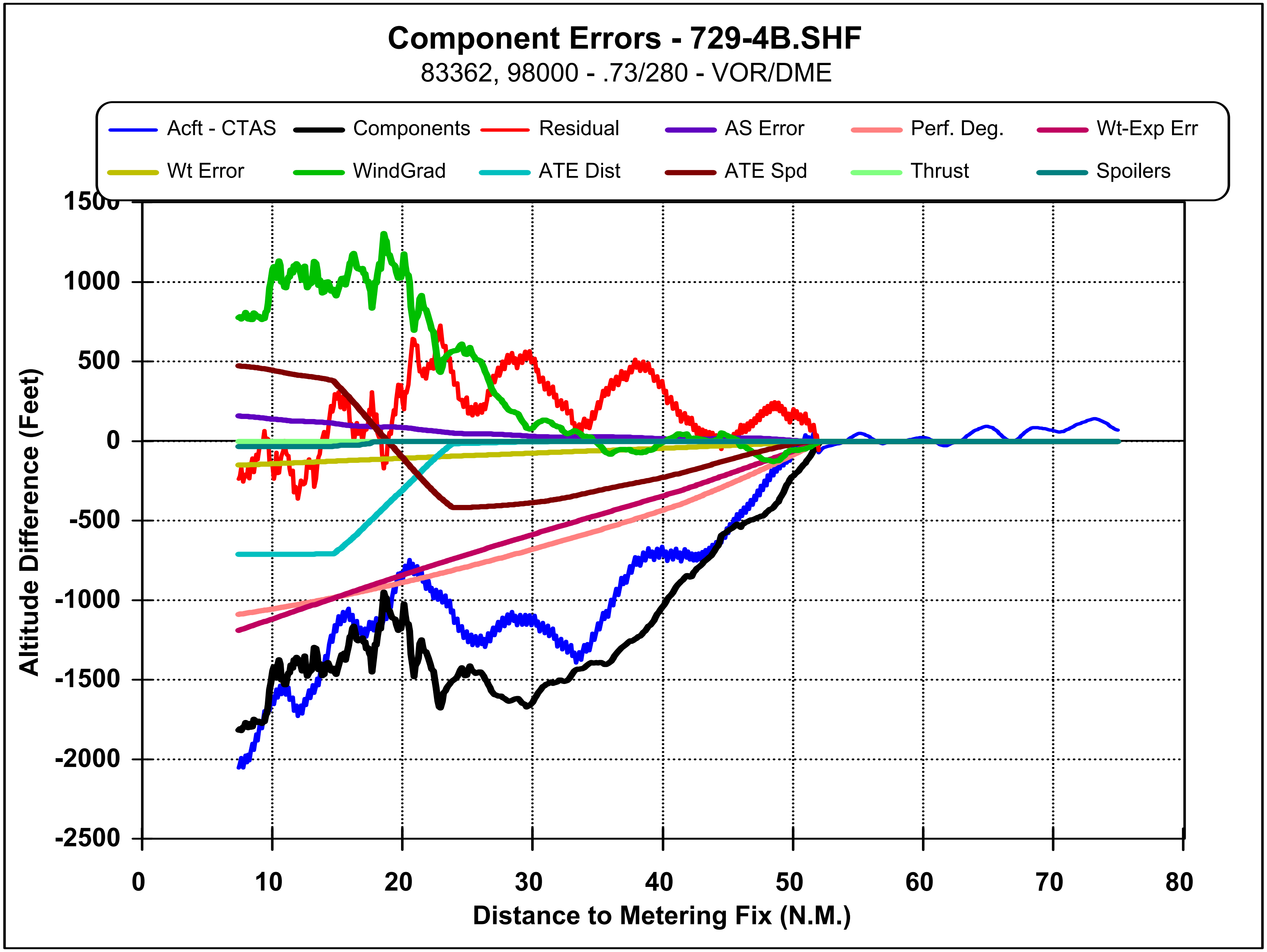
50
60
70
80
Distance to Metering Fix (NM)


Dist.-to-Go vs. Altitude - 729-4B.SHF
83362, 98000 - .73/280 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)



Component Errors - 729-4B.SHF
83362, 98000 - .73/280 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1500
1000
500
0
-500
-1000
-1500
-2000
-2500
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 733-3

Navigation: Conventional Descent with CTAS TOD

Mach/Speed Schedule: .73/280 KIAS

Aircraft Weight: 85,364 pounds

CTAS Weight: 98,000 pounds

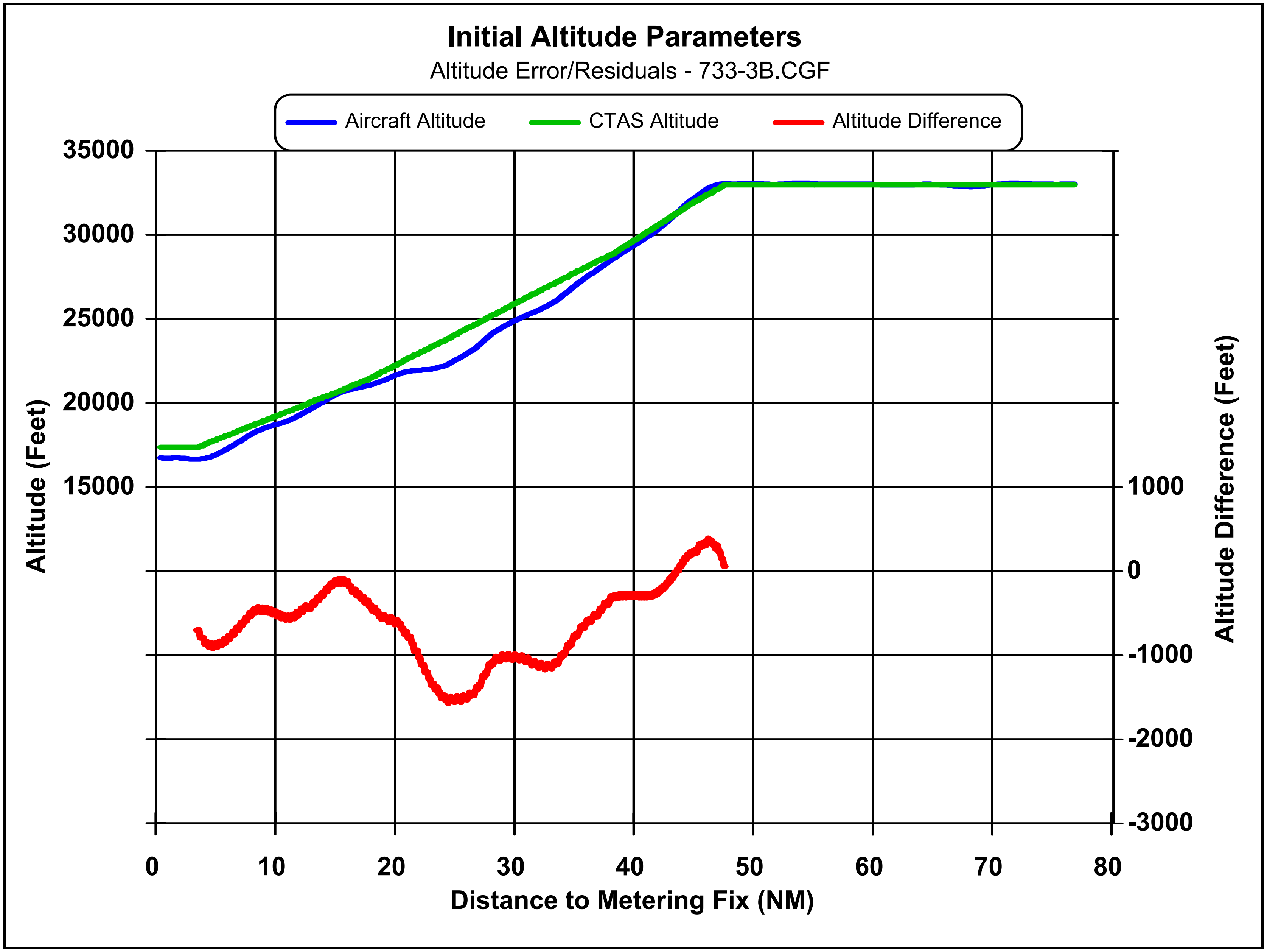
Weight Experimental Error: 13,000 pounds

Weight Error: 364 pounds

Descent Initiation Error (Time): 8 seconds

Descent Initiation Error (Distance): 0.895 NM

Vertical Descent Initiation Error (Average): 314 feet
Residual Error: -208 feet



Initial Altitude Parameters
Altitude Error/Residuals - 733-3B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
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30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
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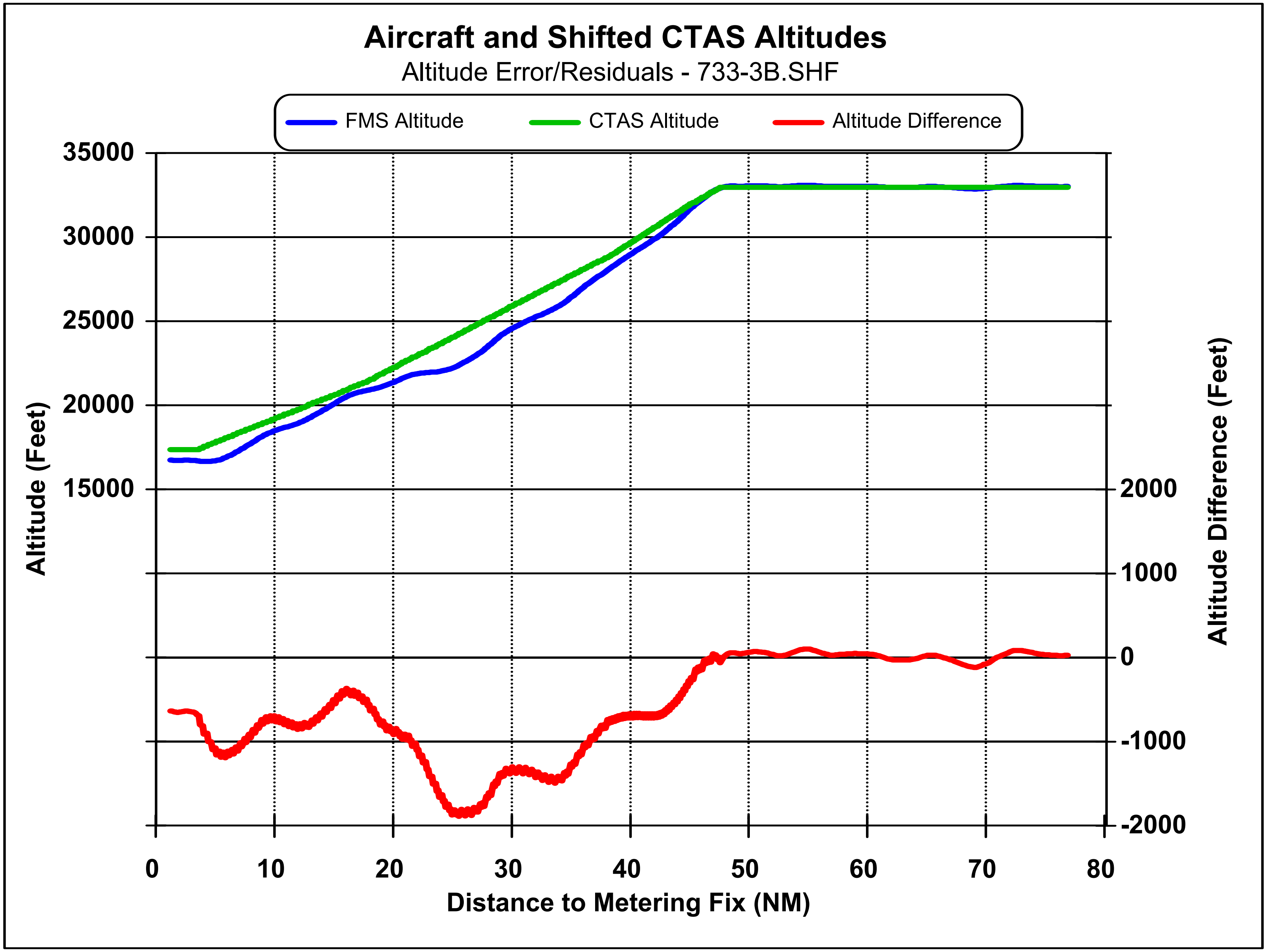
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 733-3B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
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25000
20000
15000
Altitude (Feet)
2000
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Altitude Difference (Feet)
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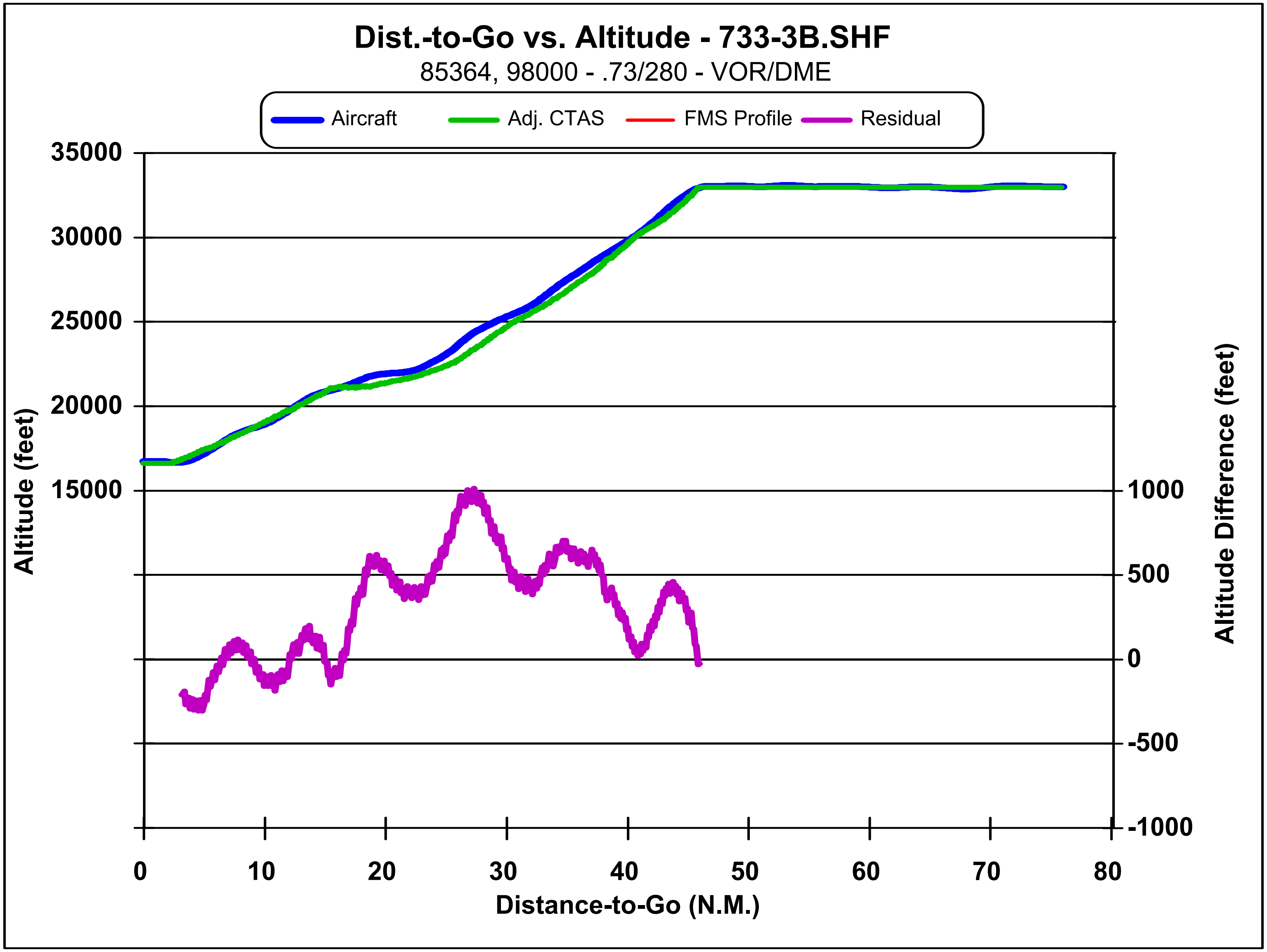
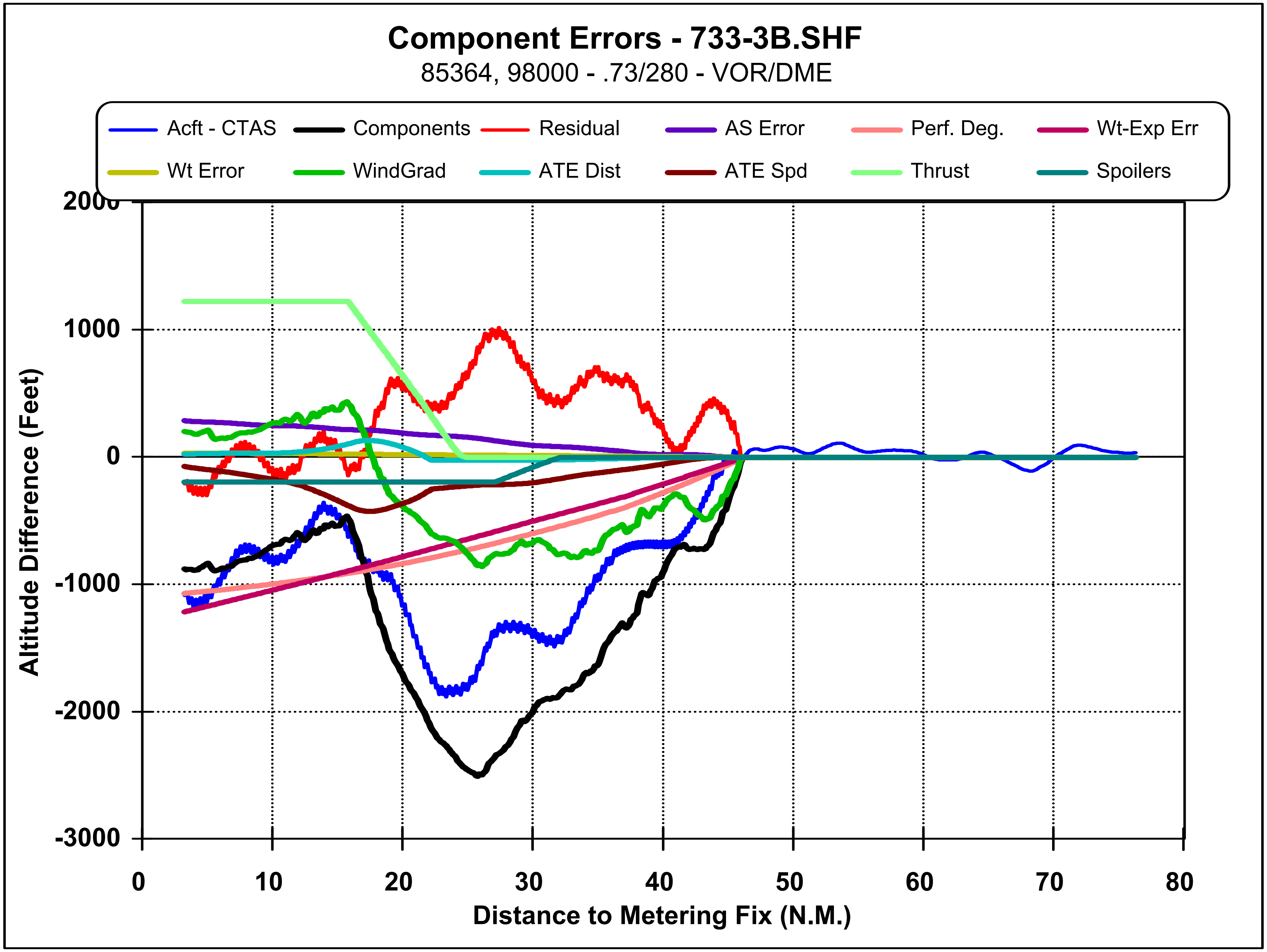
70
80
Distance to Metering Fix (NM)

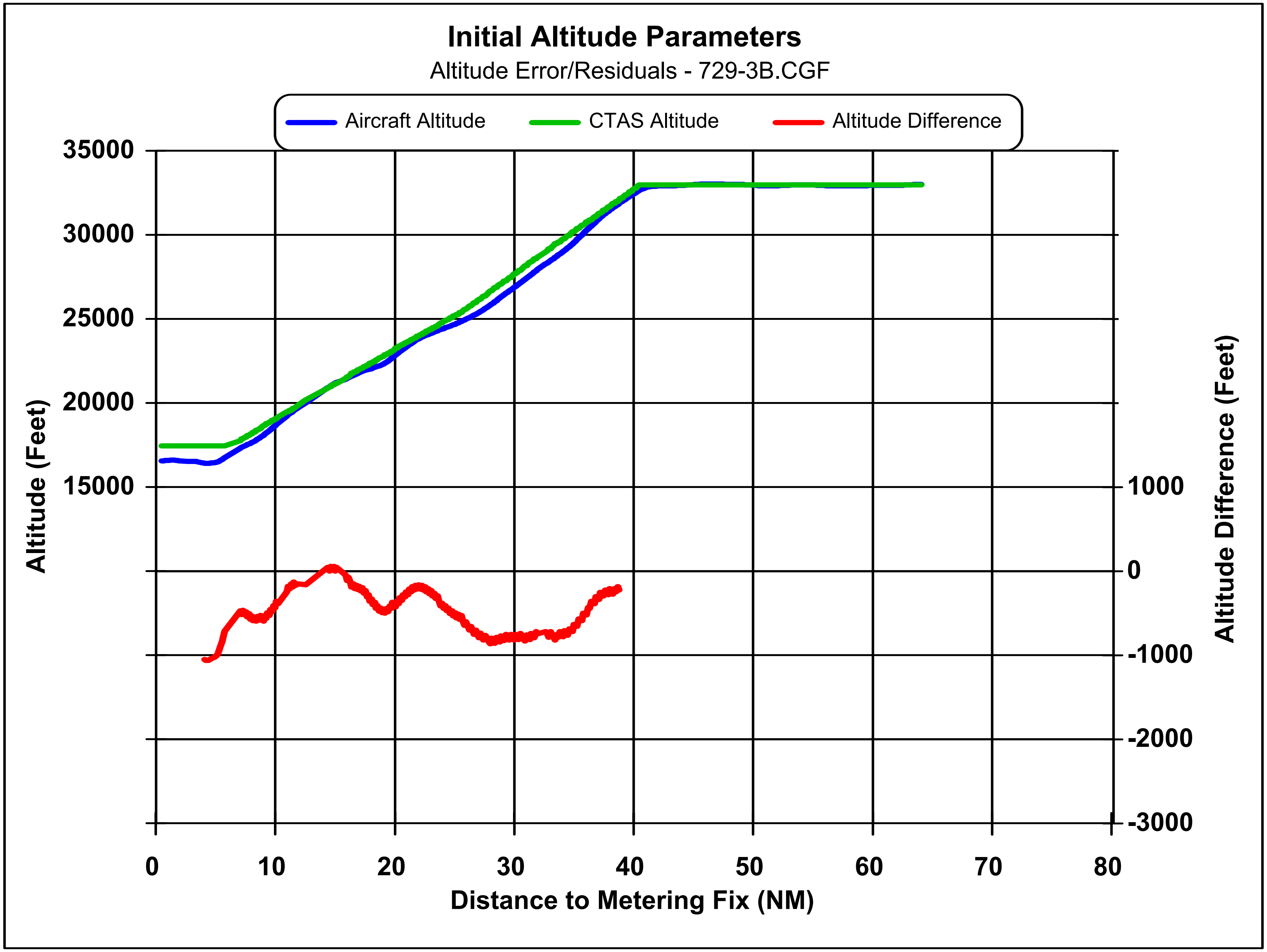


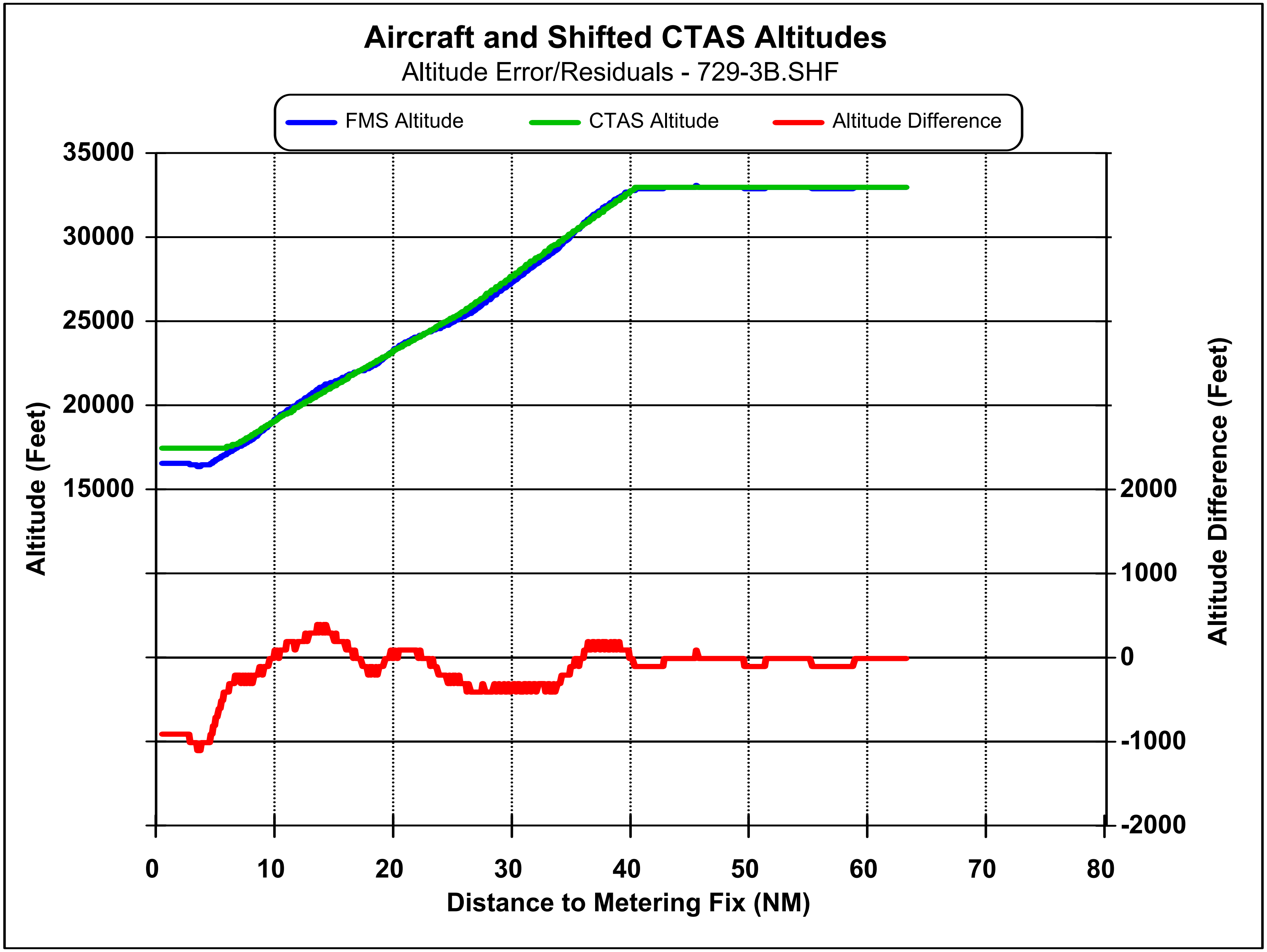
Dist.-to-Go vs. Altitude - 733-3B.SHF
85364, 98000 - .73/280 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
Altitude (feet)
35000
30000
25000
20000
15000
Altitude Difference (feet)
1000
500
0
-500
-1000
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 733-3B.SHF
85364, 98000 - .73/280 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
2000
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

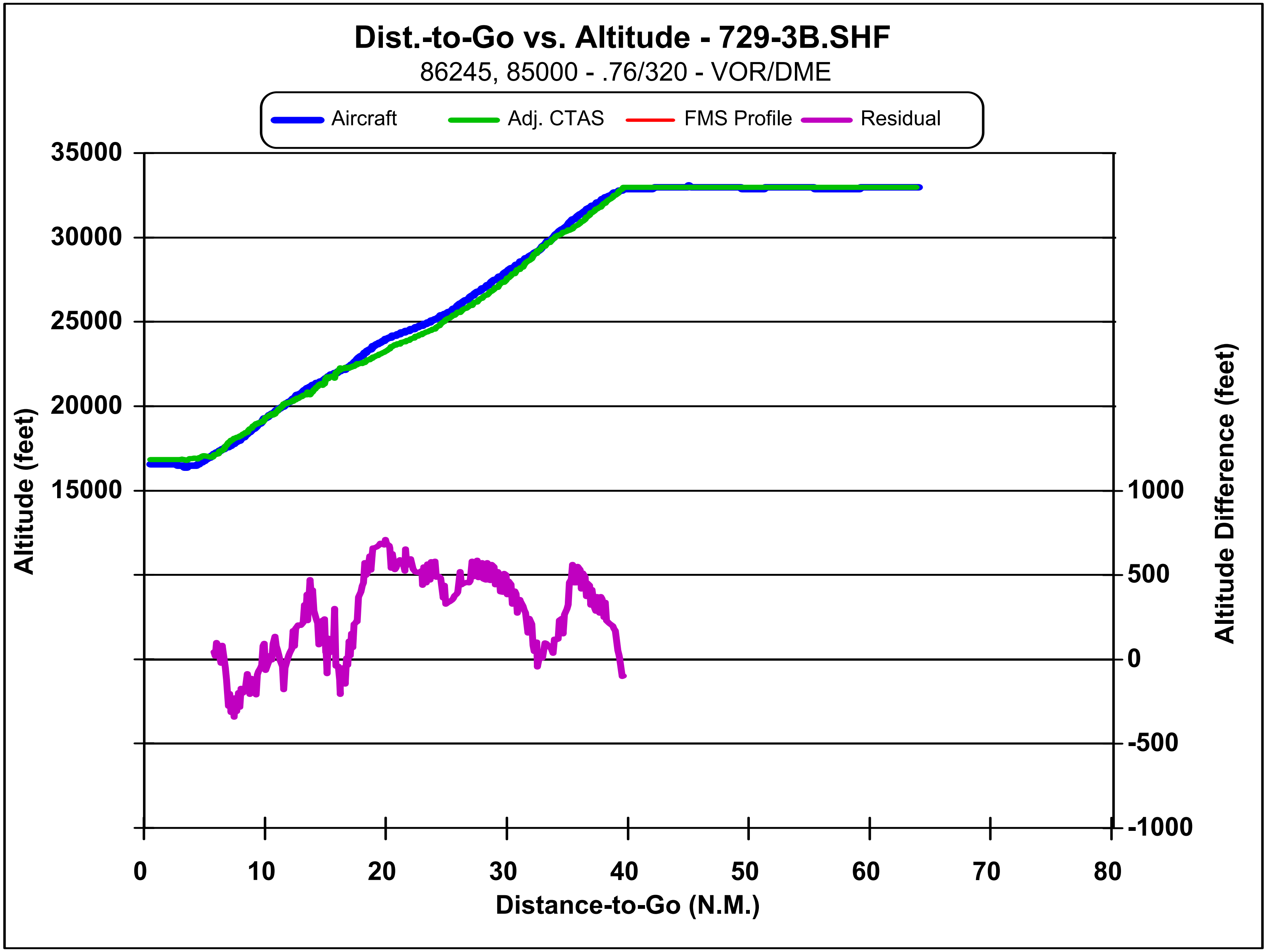
Run: 729-3

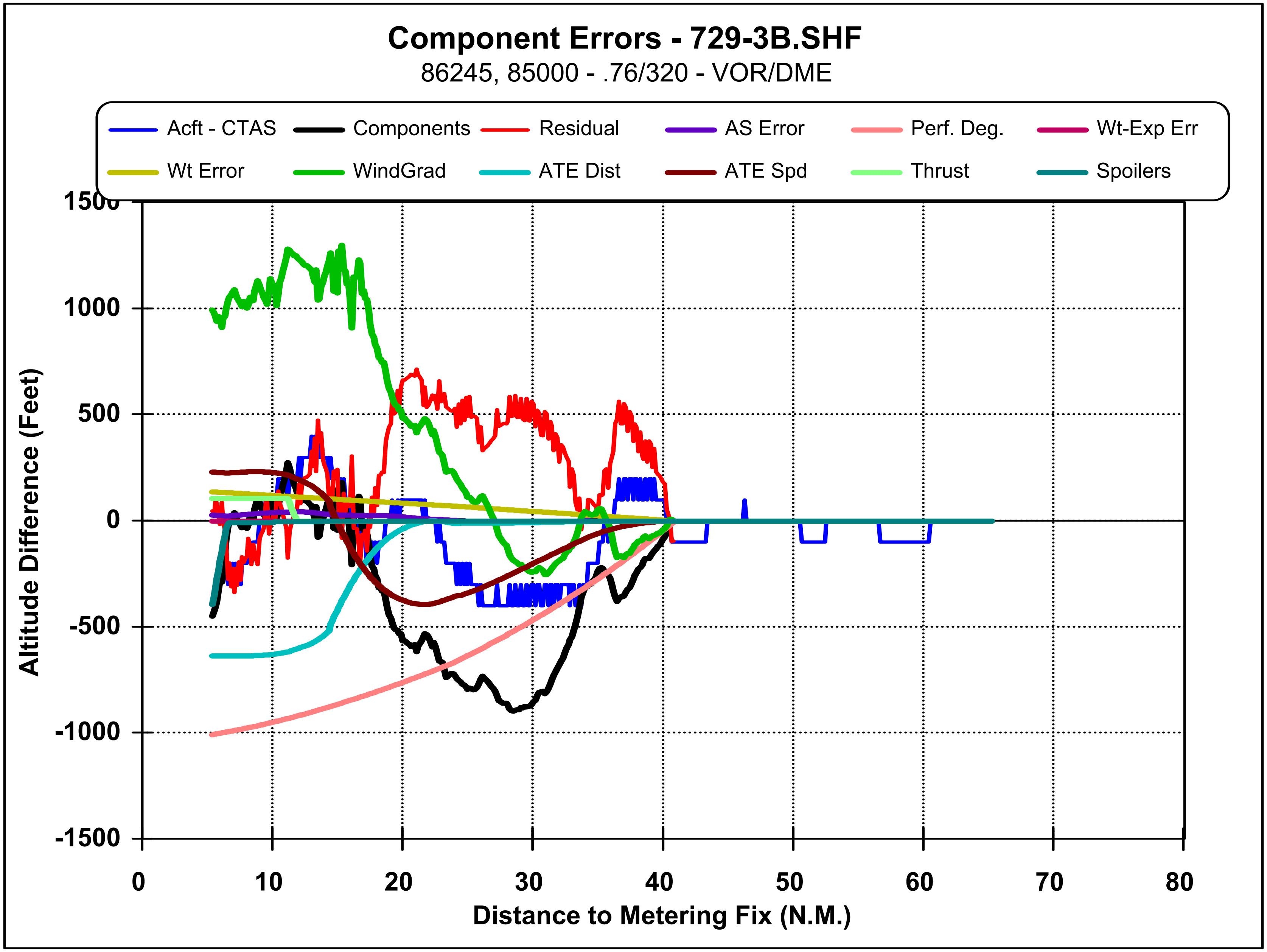
Navigation: Conventional Descent with CTAS TOD
Mach/Speed Schedule: .736/320 KIAS
Aircraft Weight: 86,245 pounds
CTAS Weight: 85,000 pounds
Weight Experimental Error: 0 pounds
Weight Error: 1,245 pounds
Descent Initiation Error (Time):   -6 seconds
Descent Initiation Error (Distance):   -0.766 NM
Vertical Descent Initiation Error (Average): -354 feet
Residual Error: 49 feet


Initial Altitude Parameters
Altitude Error/Residuals - 729-3B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 729-3B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Dist.-to-Go vs. Altitude - 729-3B.SHF
86245, 85000 - .76/320 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)



Component Errors - 729-3B.SHF
86245, 85000 - .76/320 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1500
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 729B-5

Navigation: Conventional Descent with CTAS TOD

Mach/Speed Schedule: .76/320 KIAS

Aircraft Weight: 78,586 pounds

CTAS Weight: 98,000 pounds

Weight Experimental Error: 13,000 pounds

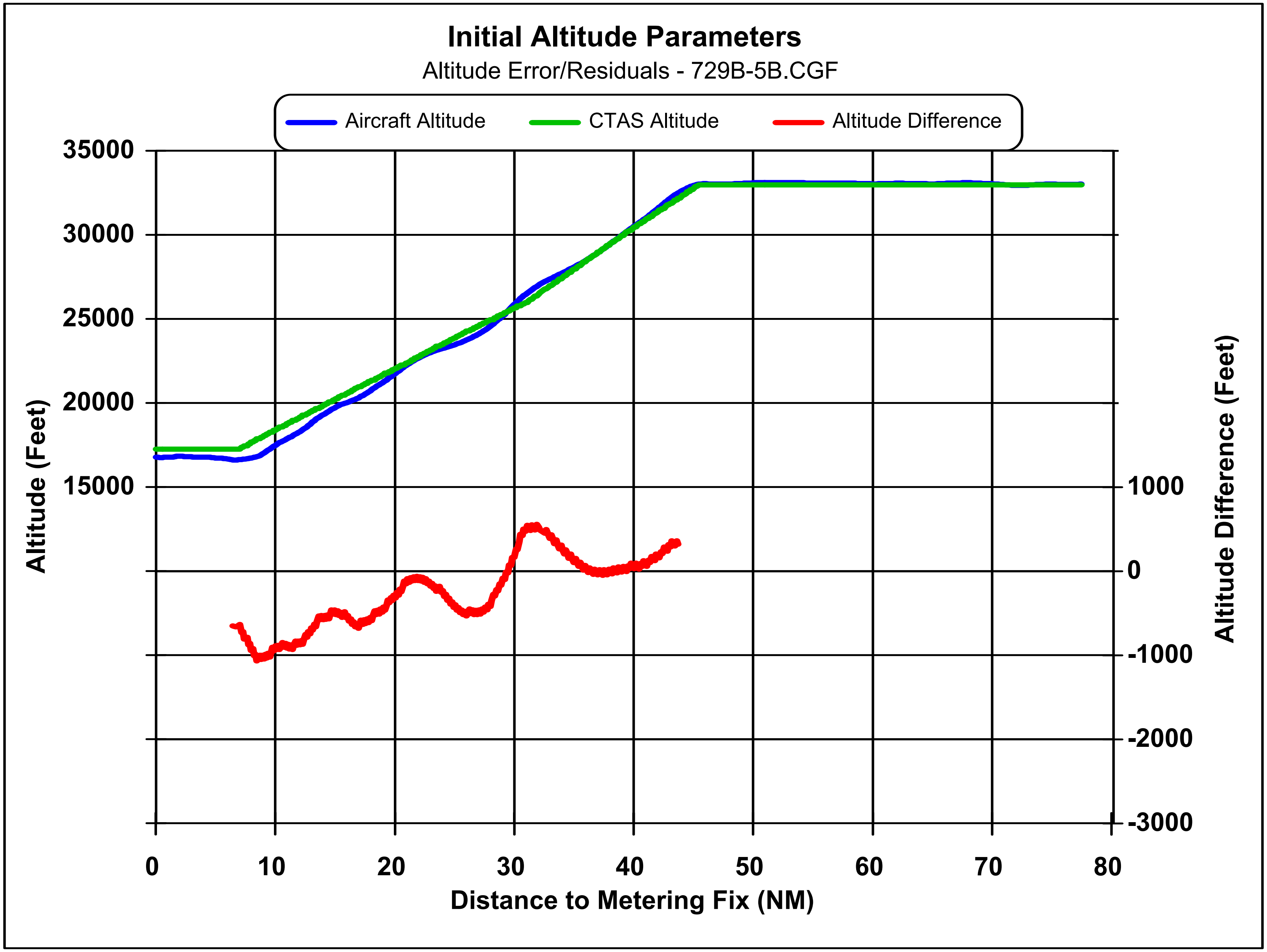
Weight Error: -6,414 pounds

Descent Initiation Error (Time):   5 seconds

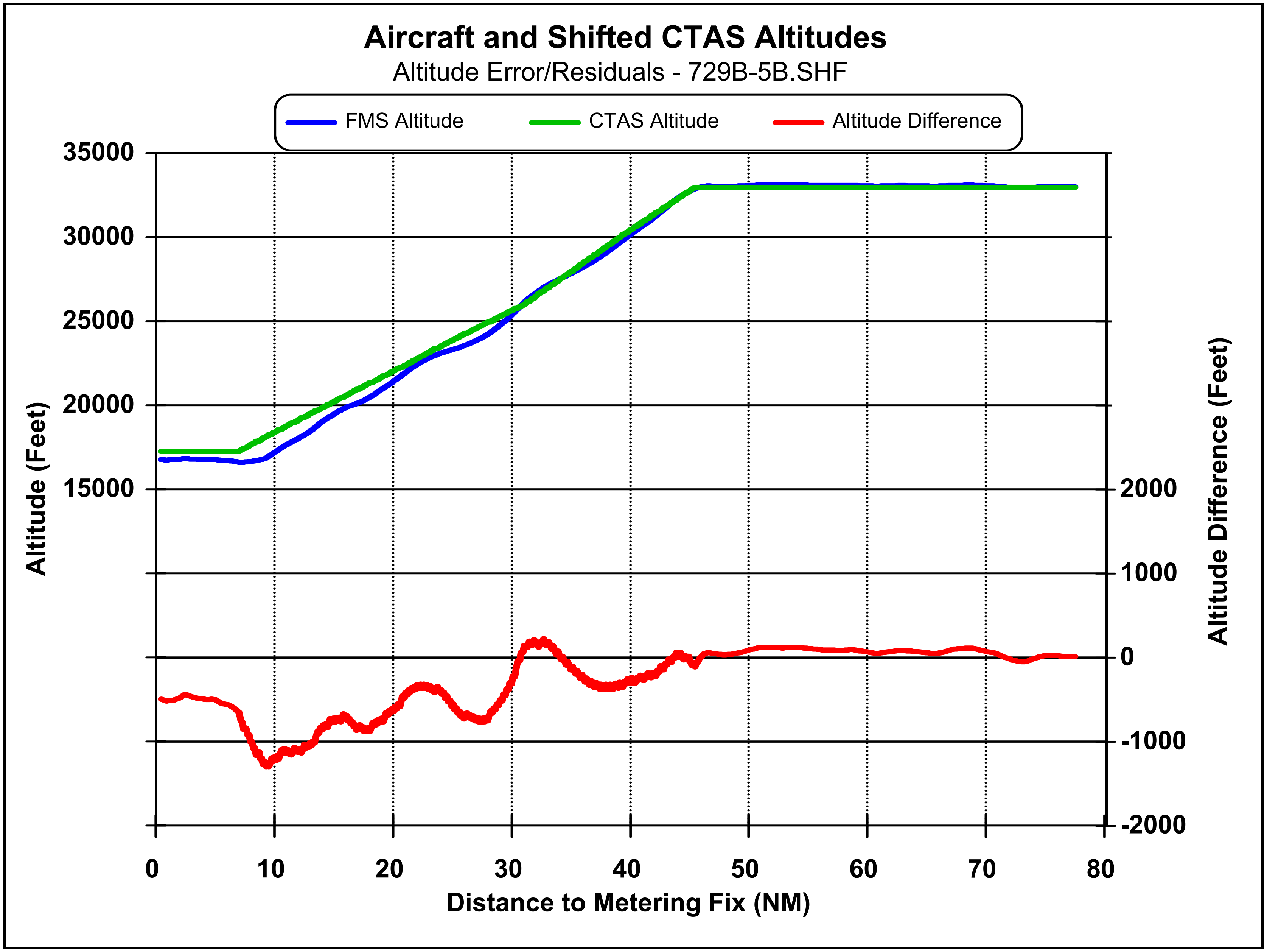
Descent Initiation Error (Distance):   0.690 NM

Vertical Descent Initiation Error (Average): 258 feet

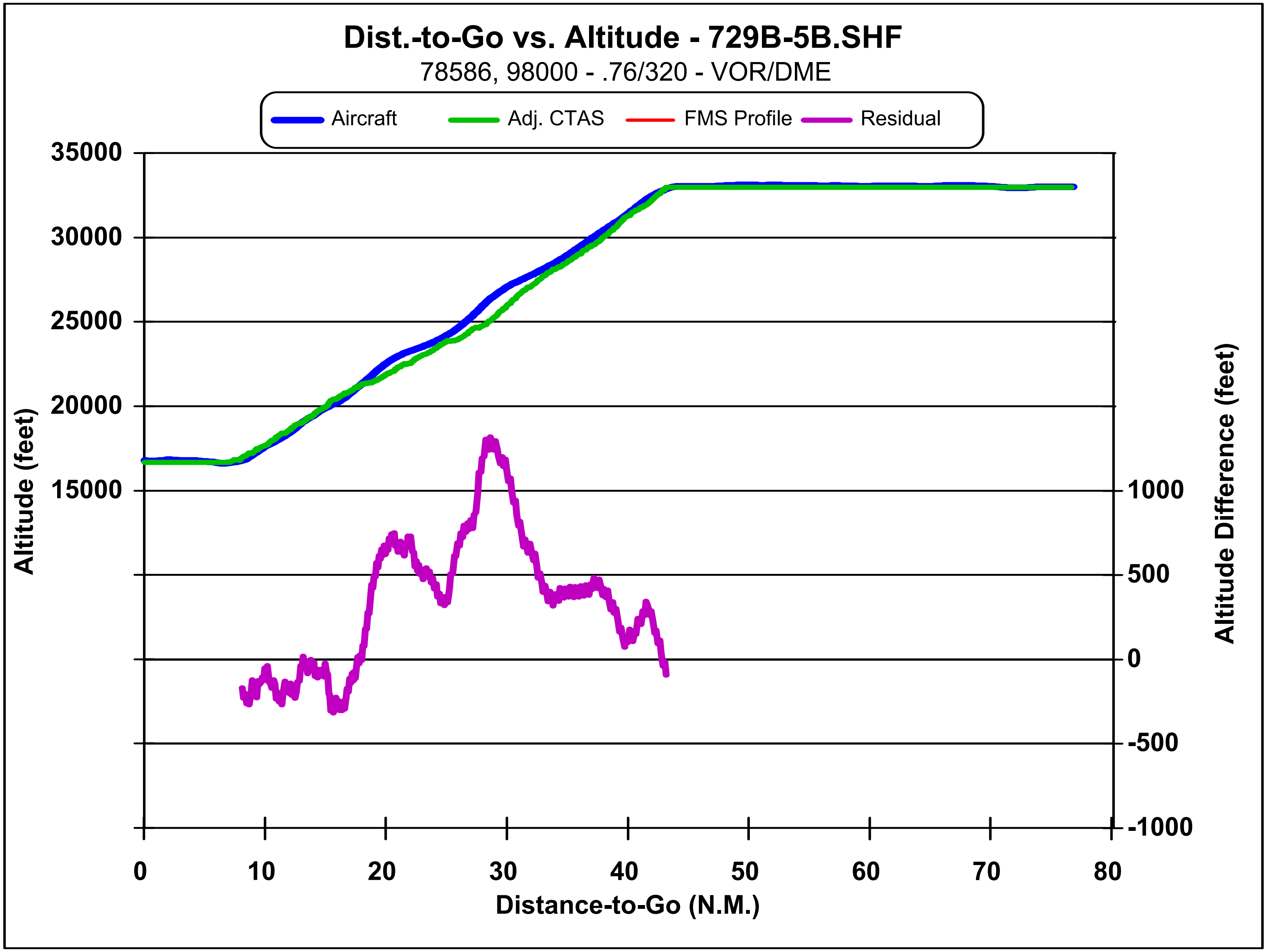
Residual Error: -163 feet

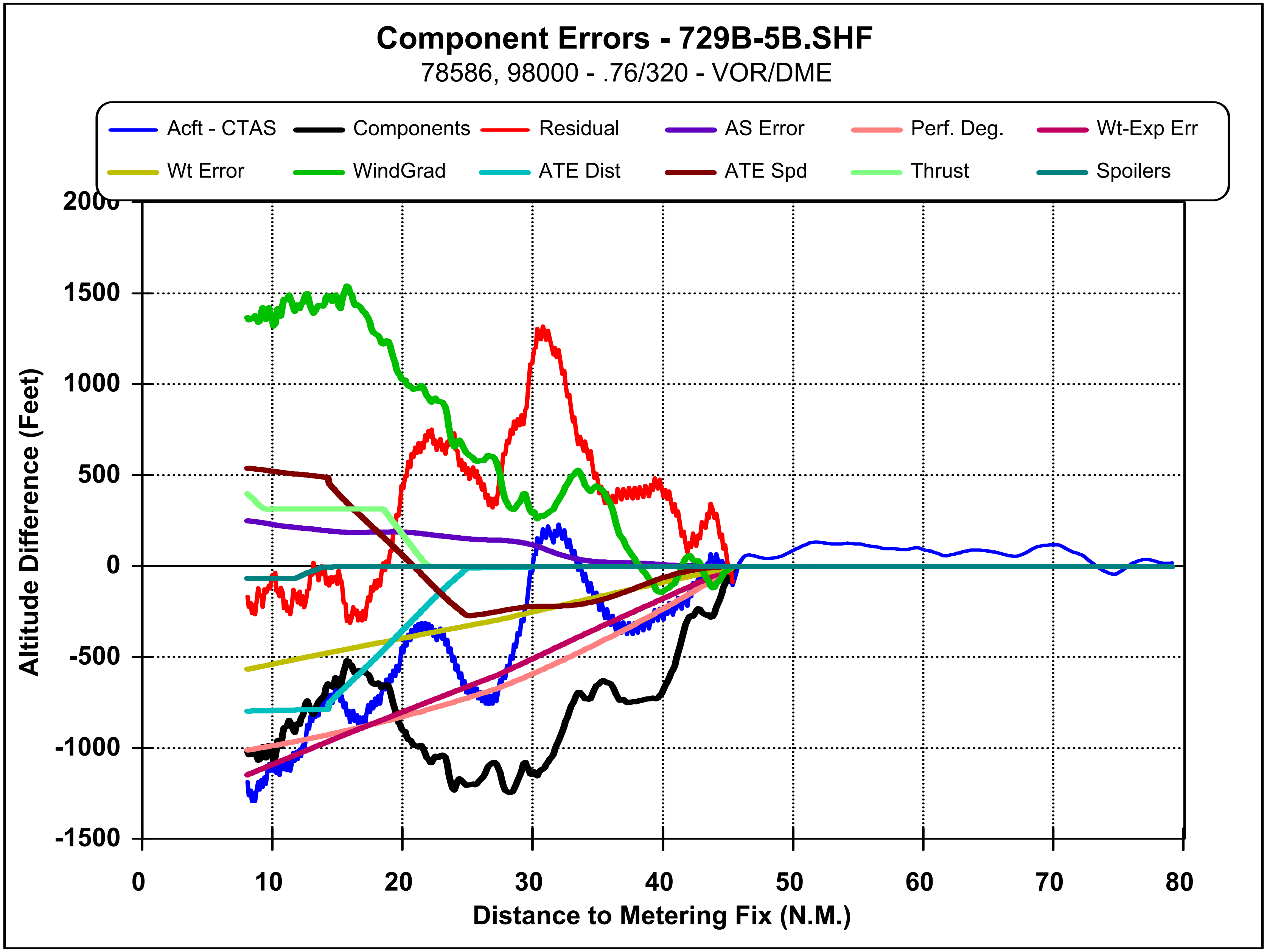


Initial Altitude Parameters
Altitude Error/Residuals - 729B-5B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
1000
0
-1000
-2000
-3000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 729B-5B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Dist.-to-Go vs. Altitude - 729B-5B.SHF
78586, 98000 - .76/320 - VOR/DME
Aircraft
Adj. CTAS
FMS Profile
Residual
Altitude (feet)
35000
30000
25000
20000
15000
Altitude Difference (feet)
1000
500
0
-500
-1000
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 729B-5B.SHF
78586, 98000 - .76/320 - VOR/DME
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
2000
1500
1000
500
0
-500
-1000
-1500
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Appendix B

VNAV Descent Graphs

Run: 729B-1

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .76/240 KIAS

Aircraft Weight: 90,084 pounds

CTAS Weight: 85,000 pounds

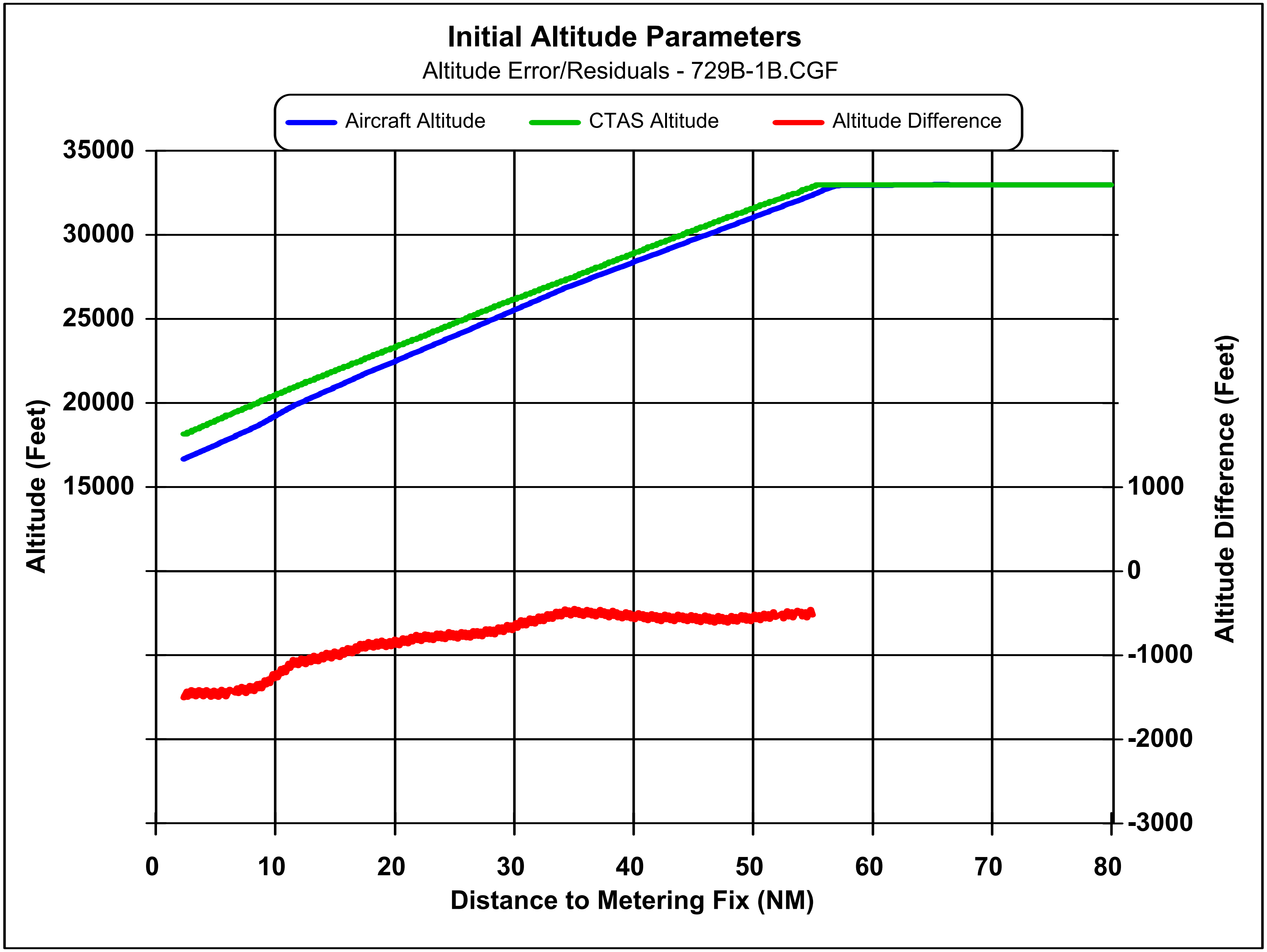
Weight Experimental Error: 0 pounds

Weight Error: 5,084 pounds

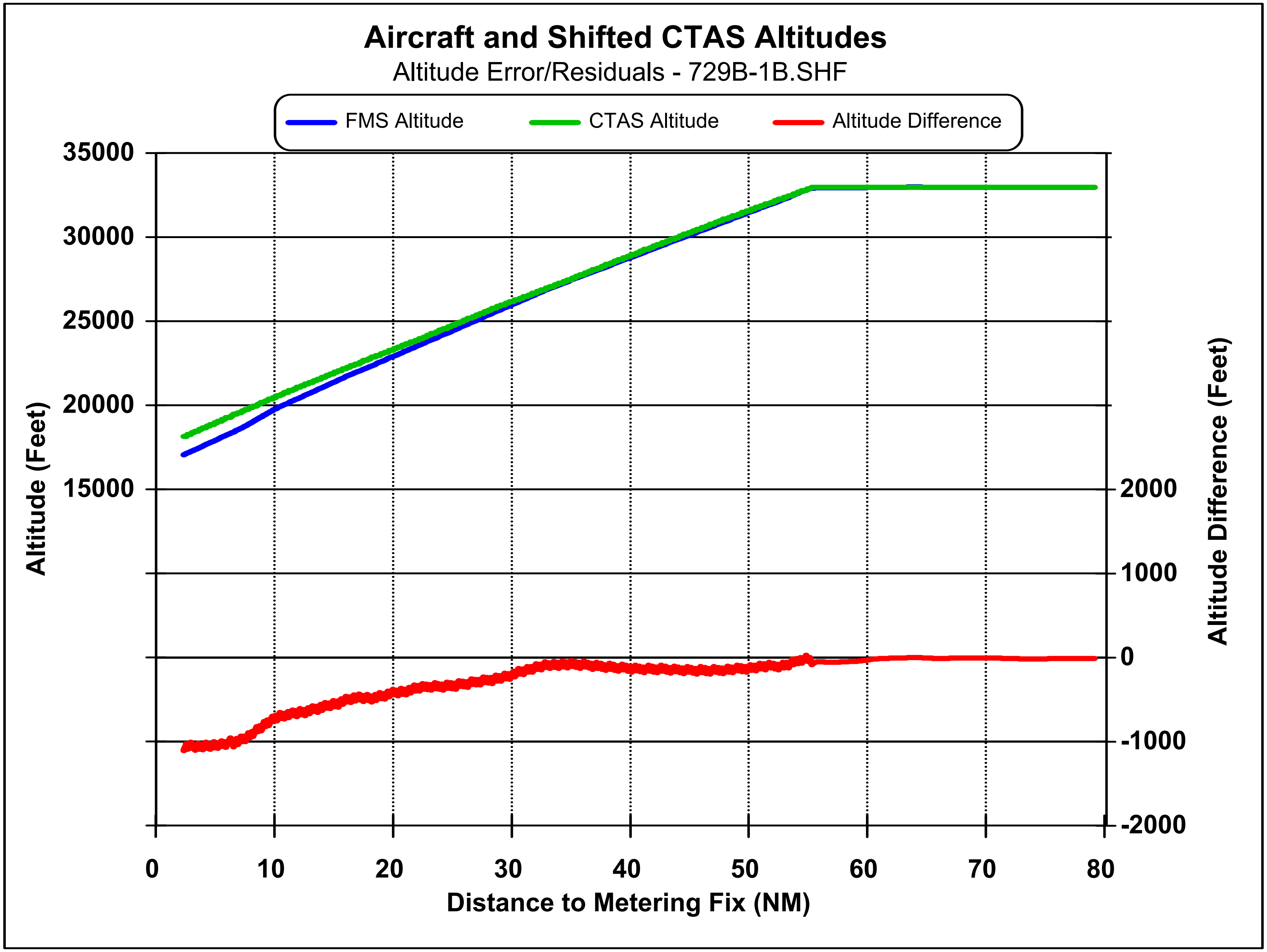
Descent Initiation Error (Time): -14 seconds

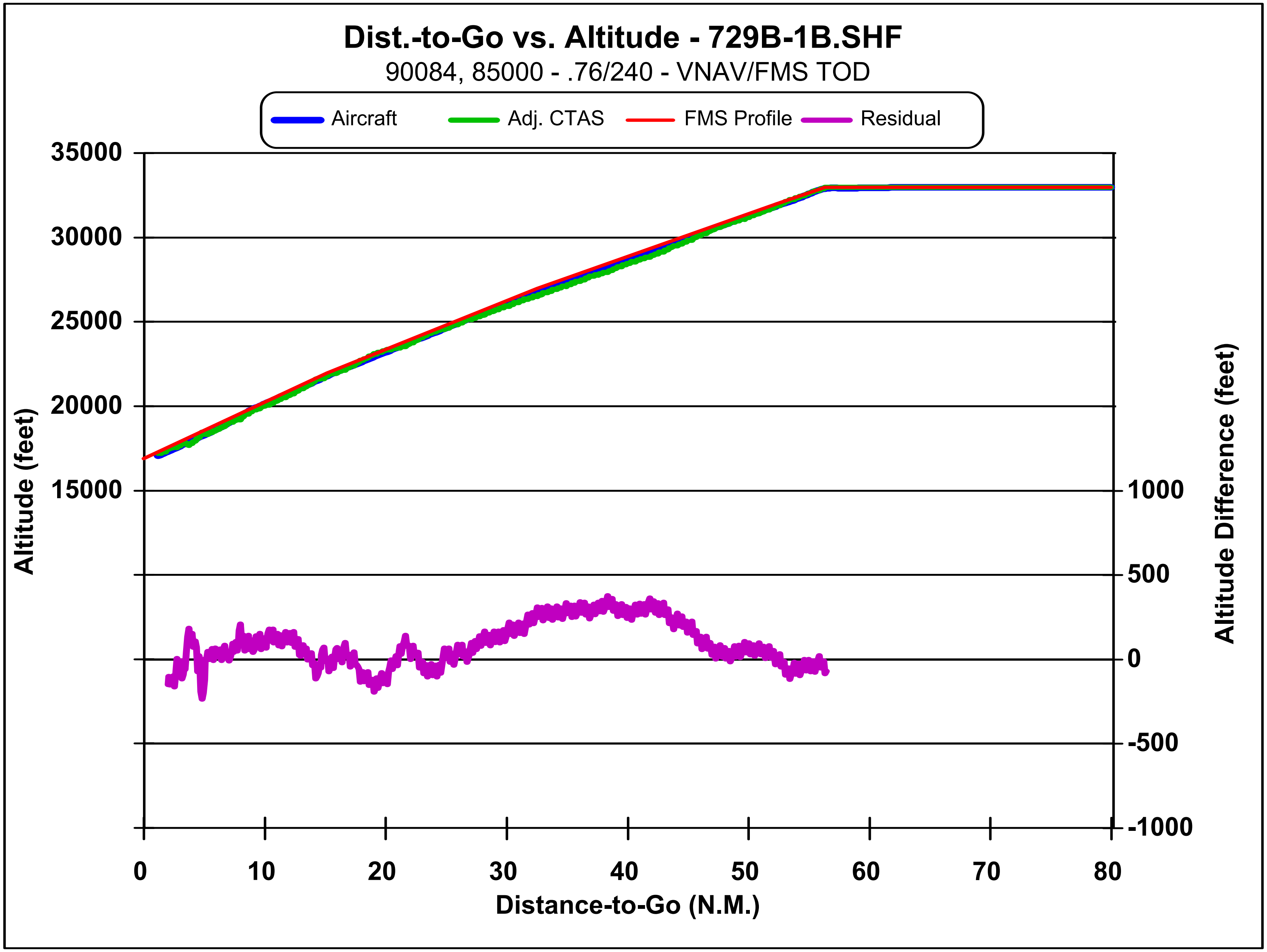
Descent Initiation Error (Distance): -1.863 NM

Vertical Descent Initiation Error (Average): -401 feet
Residual Error: -145 feet

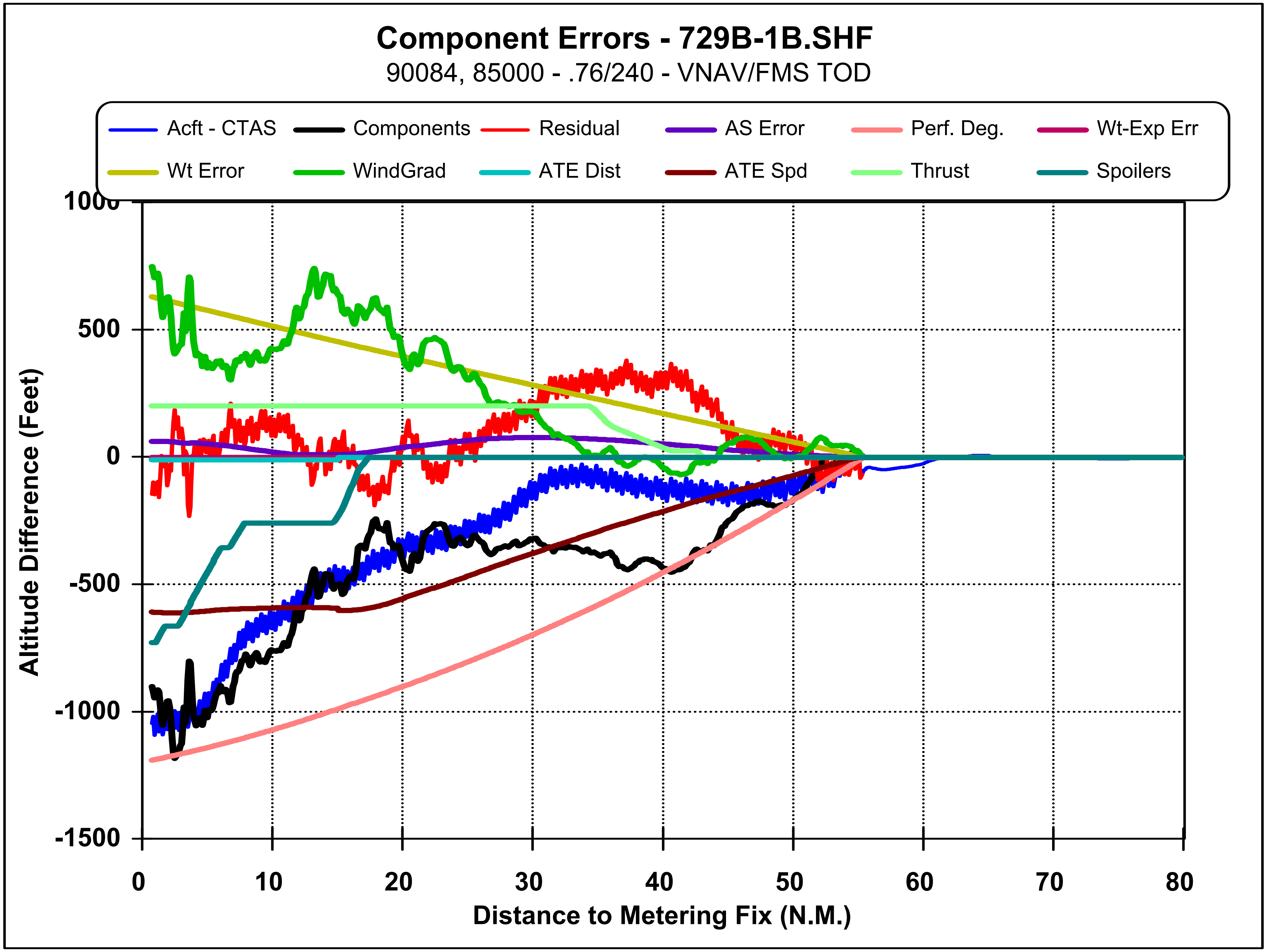


Initial Altitude Parameters
Altitude Error/Residuals - 729B-1B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 729B-1B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
2000
1000
0
-1000
-2000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 729B-1B.SHF
90084, 85000 - .76/240 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
Altitude (feet)
35000
30000
25000
20000
15000
Altitude Difference (feet)
1000
500
0
-500
-1000
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 729B-1B.SHF
90084, 85000 - .76/240 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 732-3

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .76/240 KIAS

Aircraft Weight: 84,900 pounds

CTAS Weight: 98,000 pounds

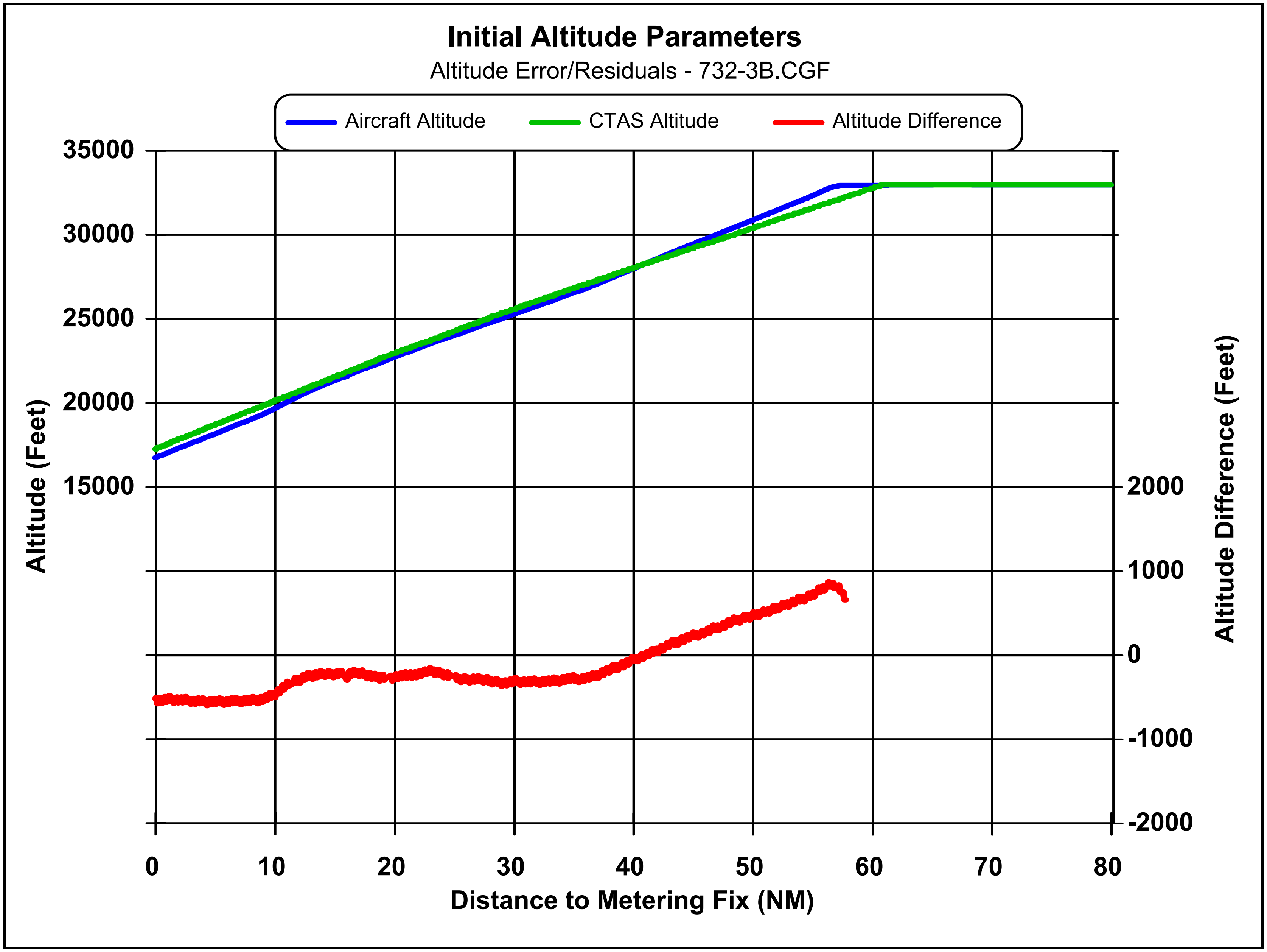
Weight Experimental Error: 13,000 pounds

Weight Error: -100 pounds

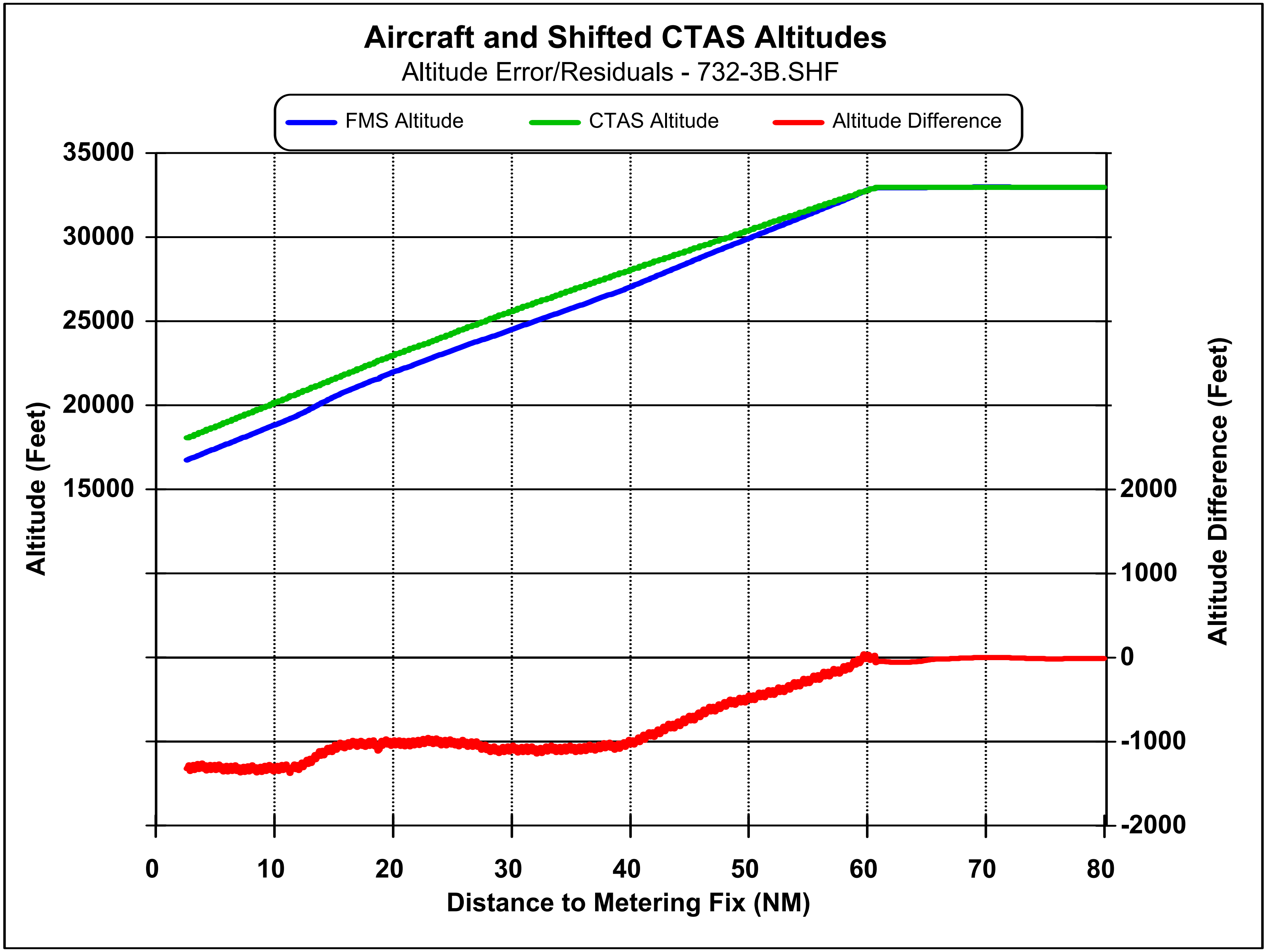
Descent Initiation Error (Time):   29 seconds

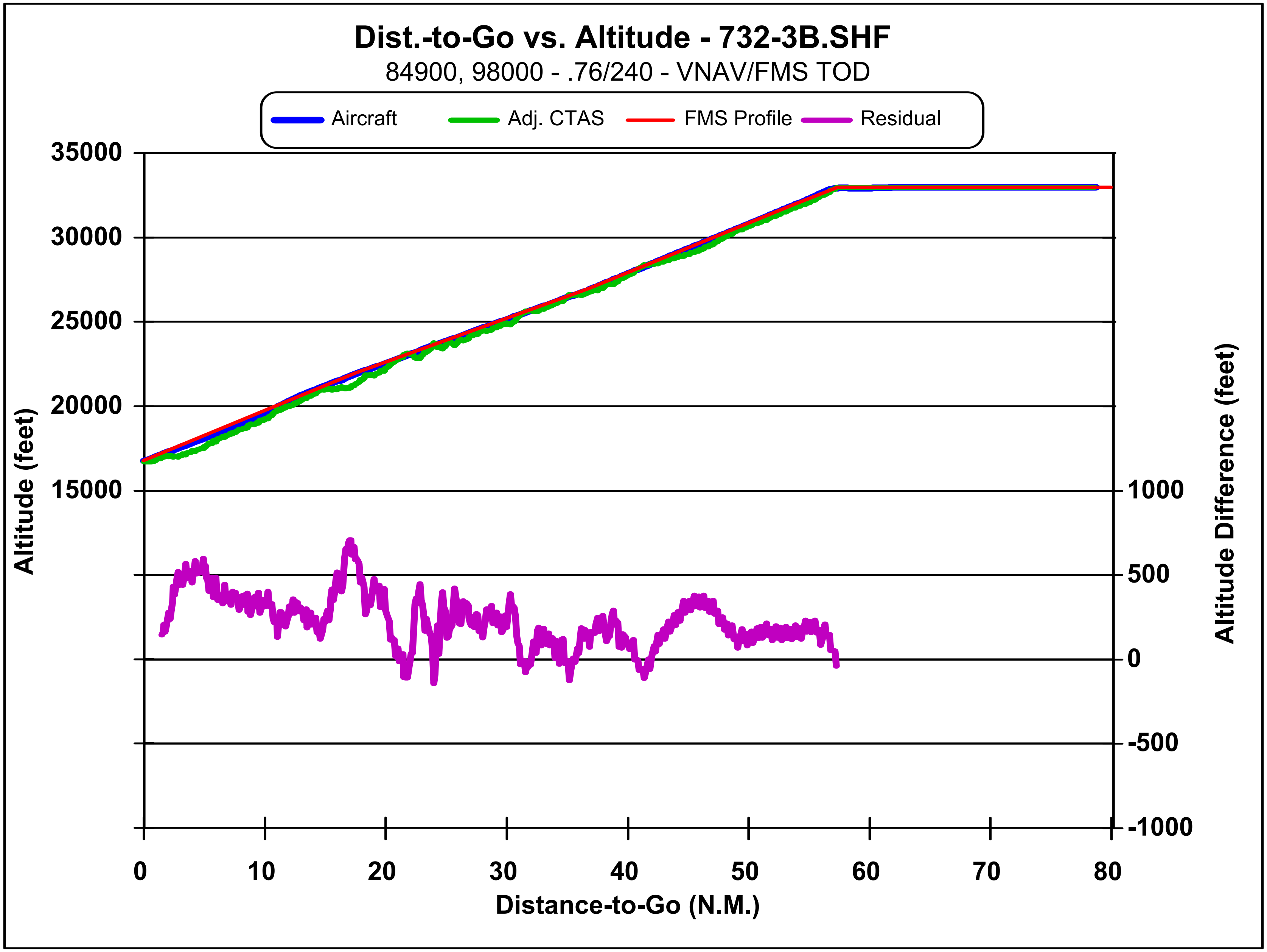
Descent Initiation Error (Distance):   3.705 NM

Vertical Descent Initiation Error (Average): 771 feet
Residual Error: 161 feet

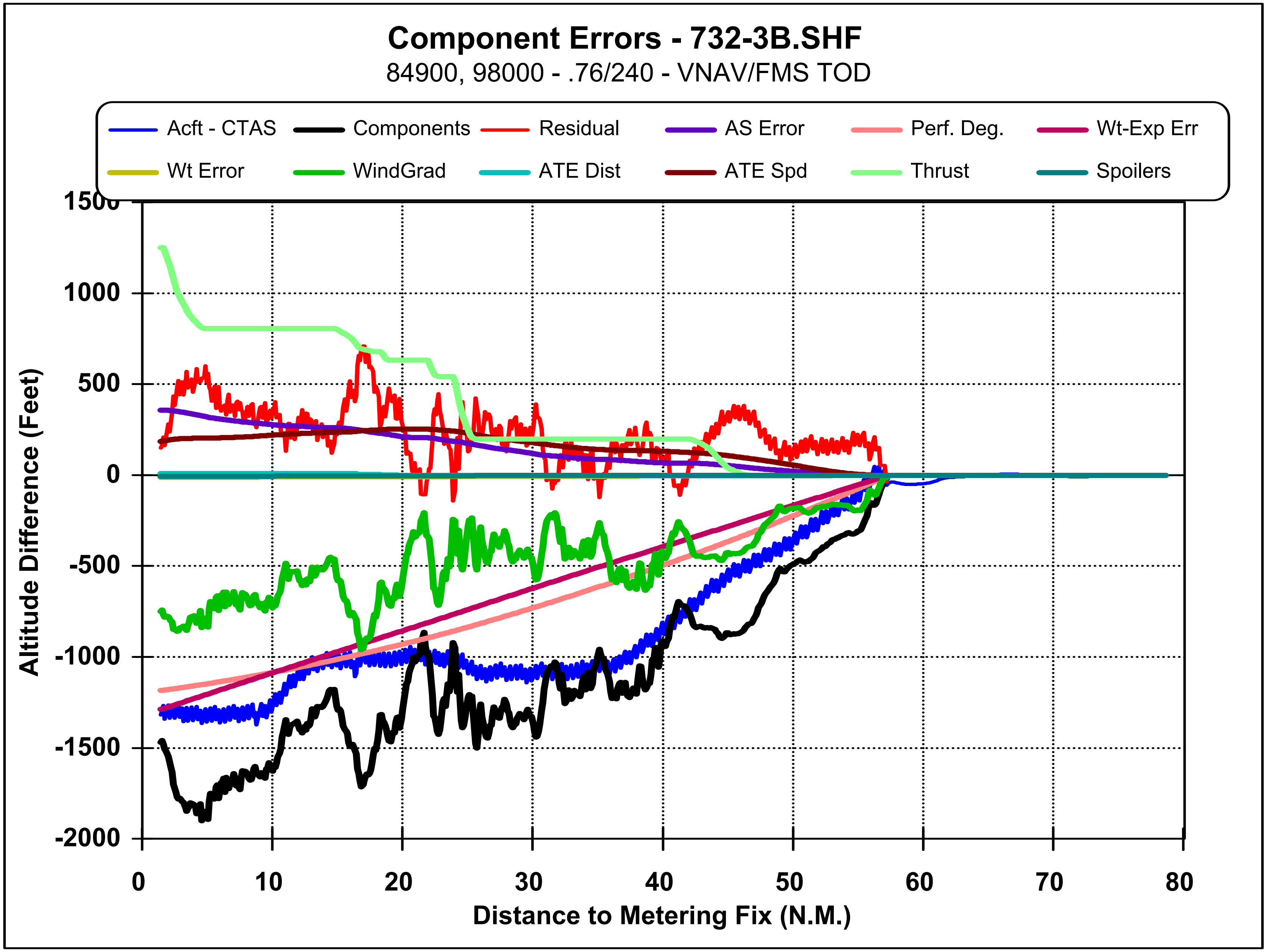


Initial Altitude Parameters
Altitude Error/Residuals - 732-3B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
2000
1000
0
-1000
-2000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 732-3B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 732-3B.SHF
84900, 98000 - .76/240 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)



Component Errors - 732-3B.SHF
84900, 98000 - .76/240 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1500
1000
500
0
-500
-1000
-1500
-2000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 733-4

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .76/240 KIAS

Aircraft Weight: 82,373 pounds

CTAS Weight: 98,000 pounds

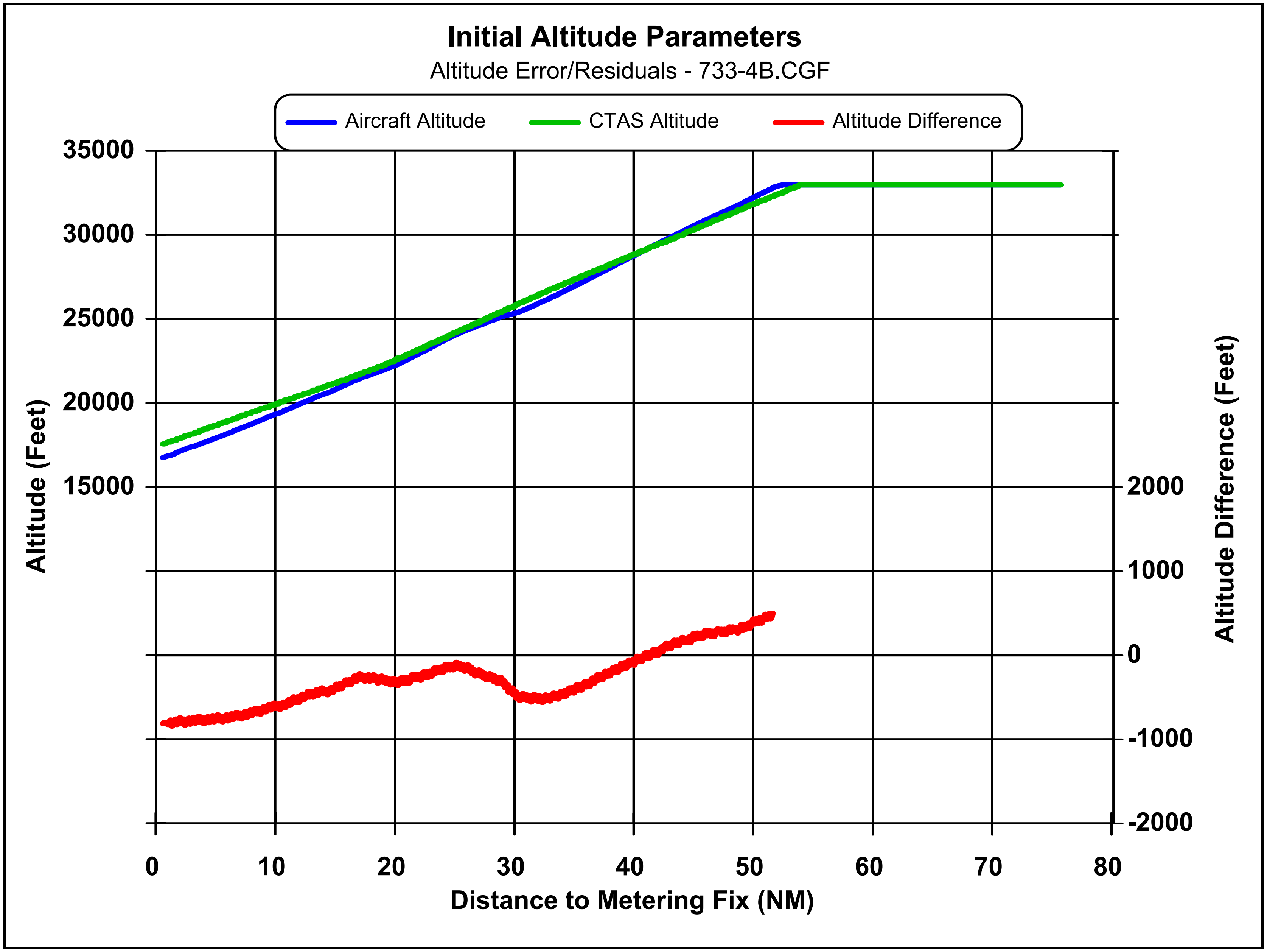
Weight Experimental Error: 13,000 pounds

Weight Error: -2,627 pounds

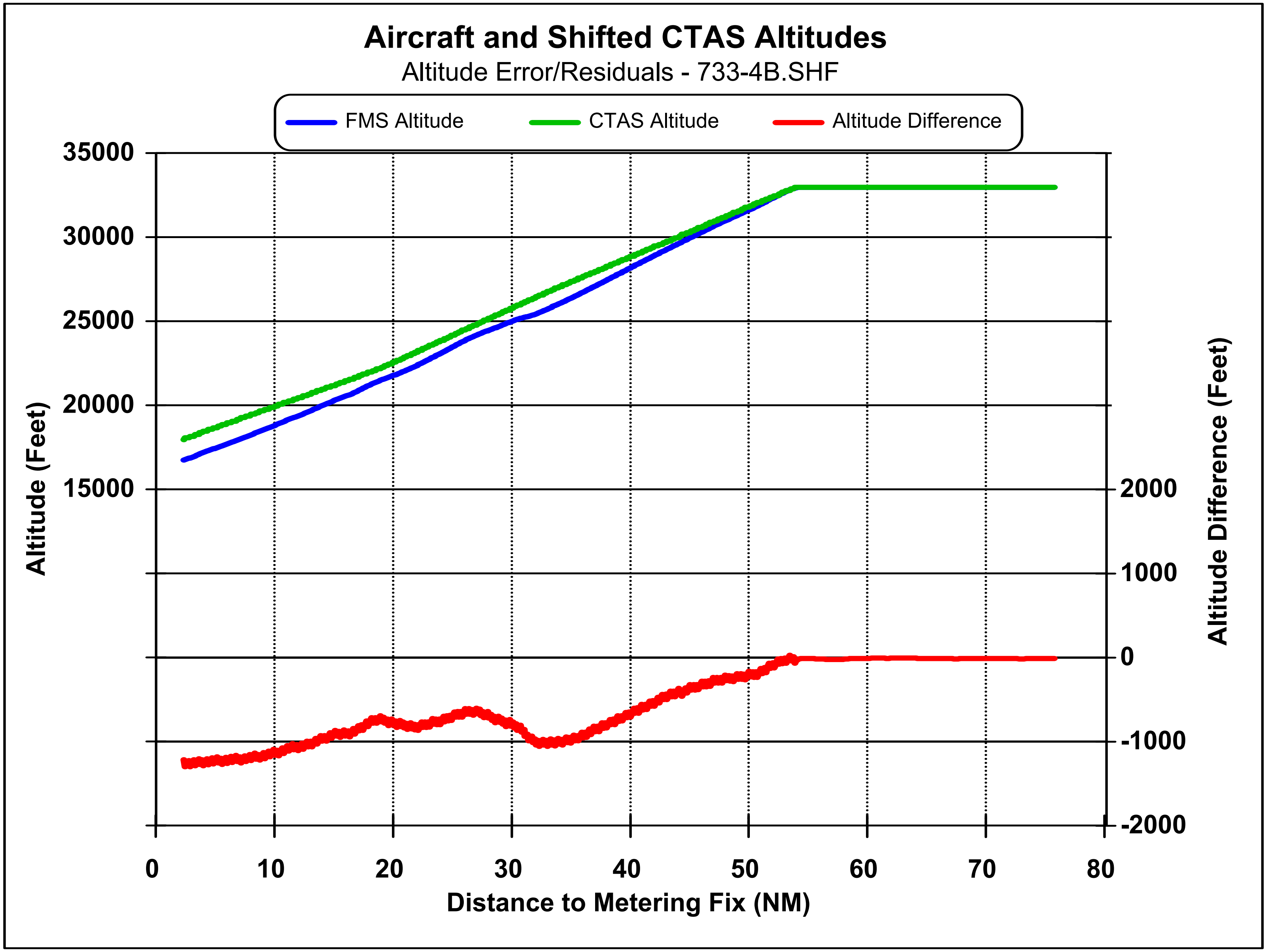
Descent Initiation Error (Time): 17 seconds

Descent Initiation Error (Distance): 1.914 NM

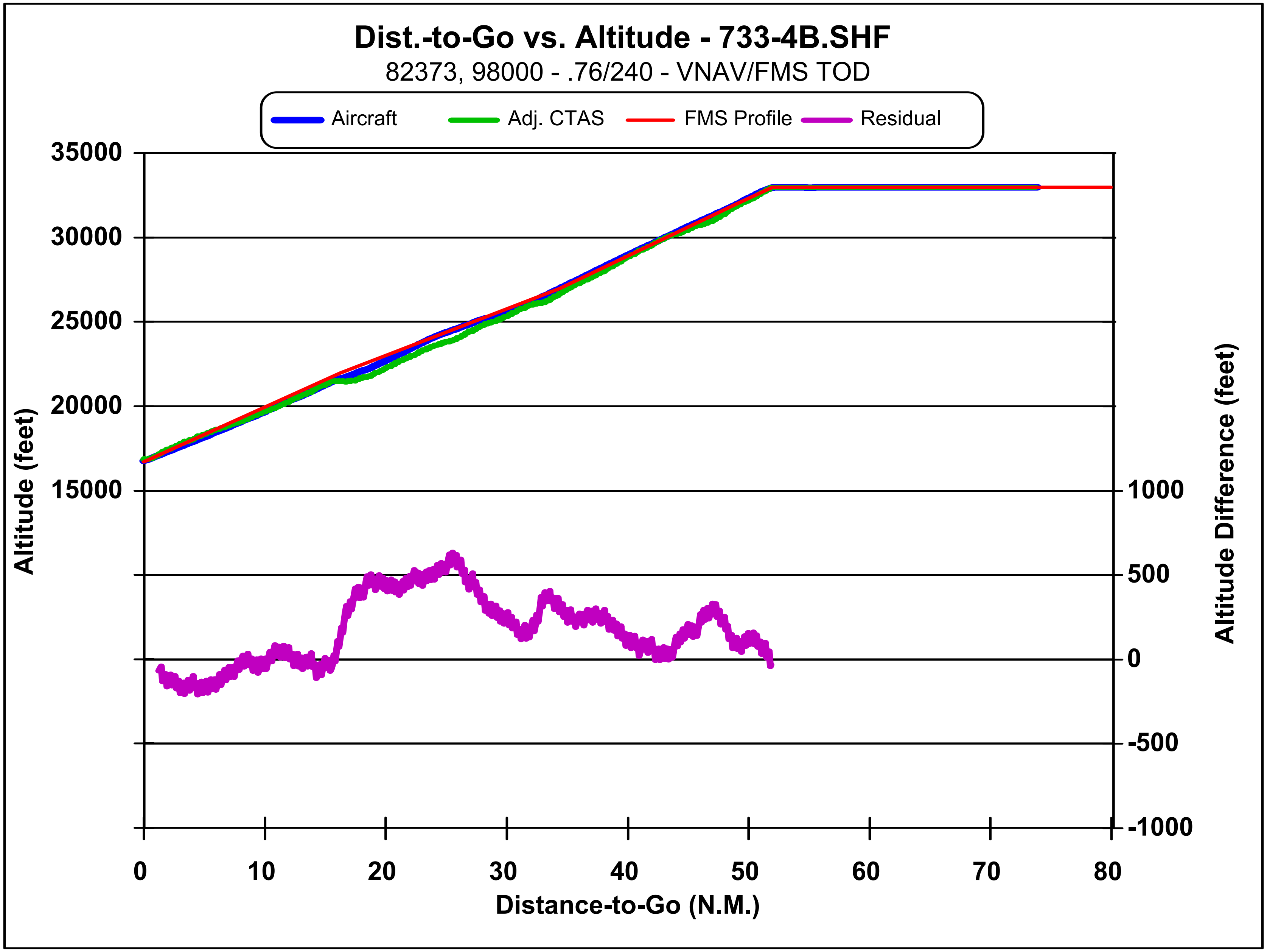
Vertical Descent Initiation Error (Average): 481 feet
Residual Error: -44 feet

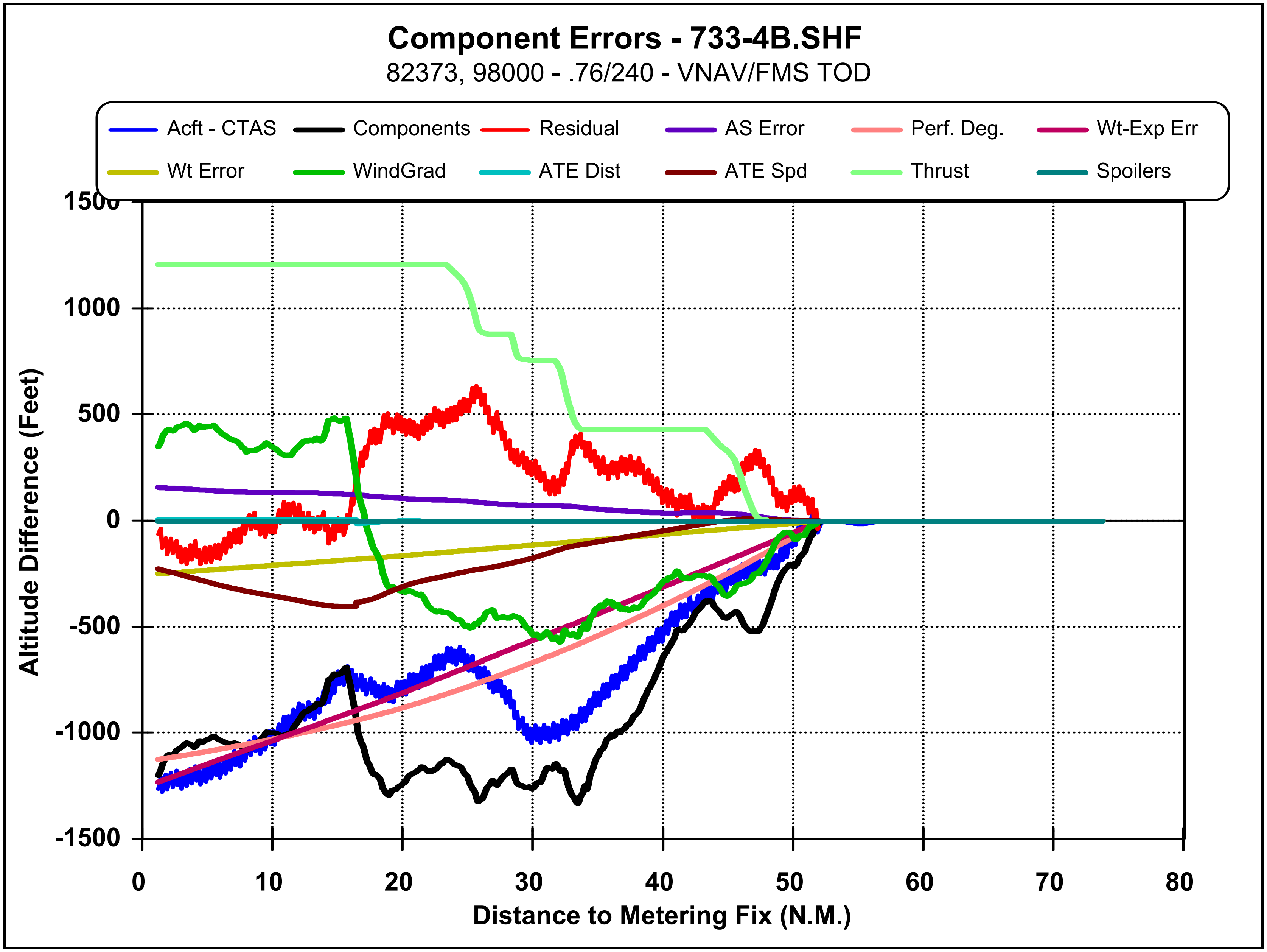


Initial Altitude Parameters
Altitude Error/Residuals - 733-4B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 733-4B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Dist.-to-Go vs. Altitude - 733-4B.SHF
82373, 98000 - .76/240 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
Altitude (feet)
35000
30000
25000
20000
15000
Altitude Difference (feet)
1000
500
0
-500
-1000
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 733-4B.SHF
82373, 98000 - .76/240 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1500
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 728-2

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .73/280 KIAS

Aircraft Weight: 86,839 pounds

CTAS Weight: 98,000 pounds

Weight Experimental Error: 13,000 pounds

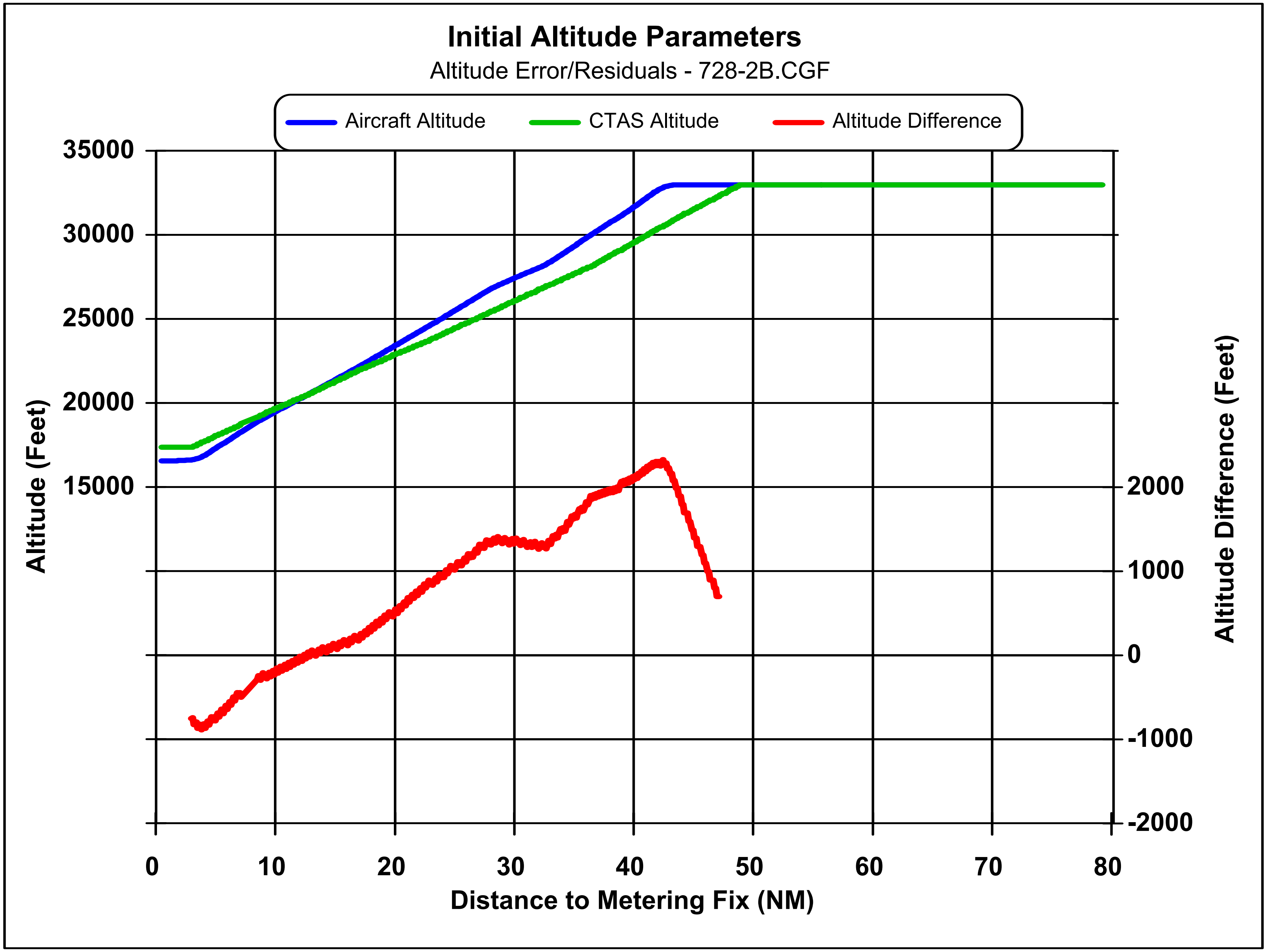
Weight Error: 1,839 pounds

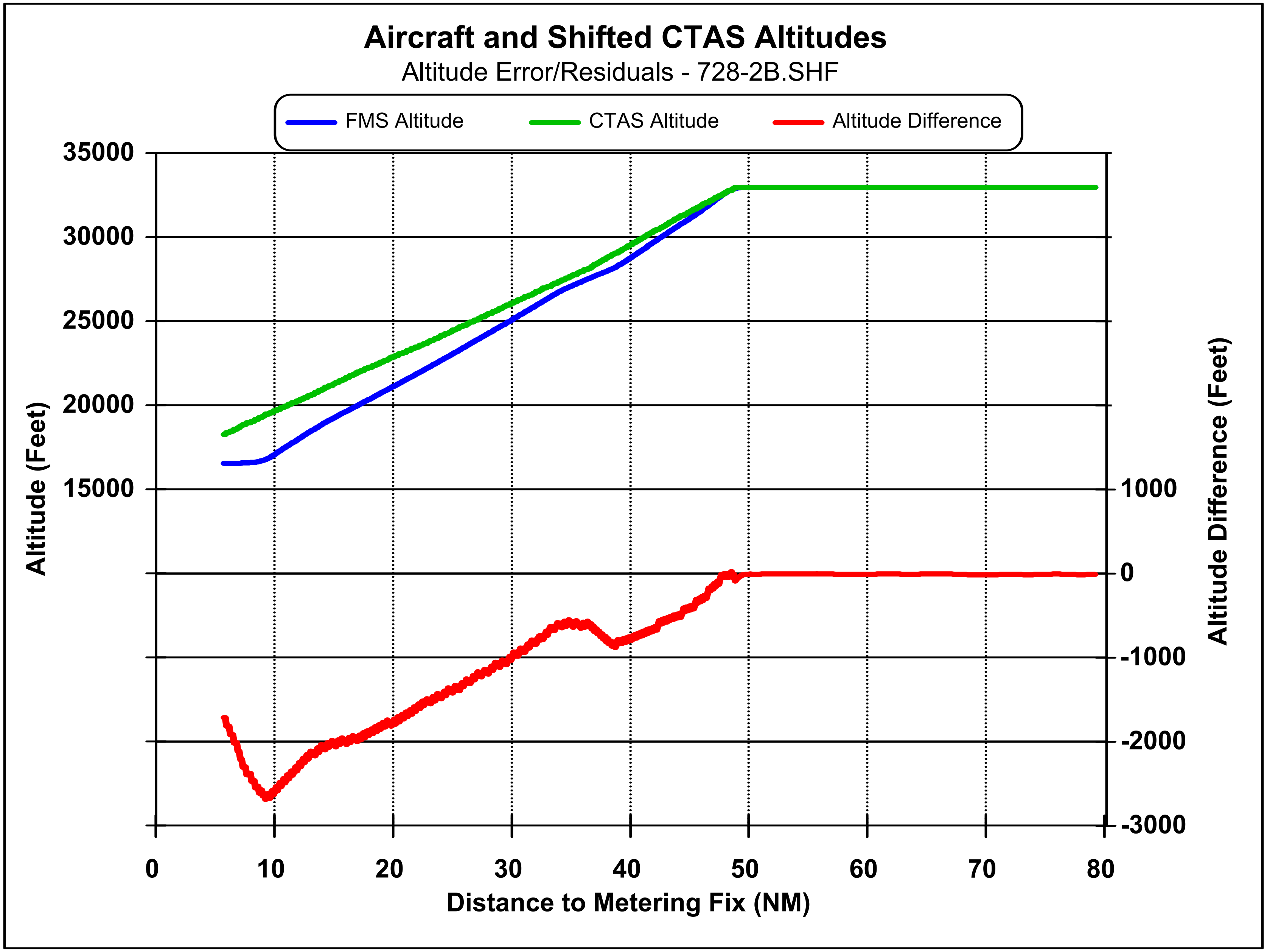
Descent Initiation Error (Time):   51 seconds

Descent Initiation Error (Distance):   6.156 NM

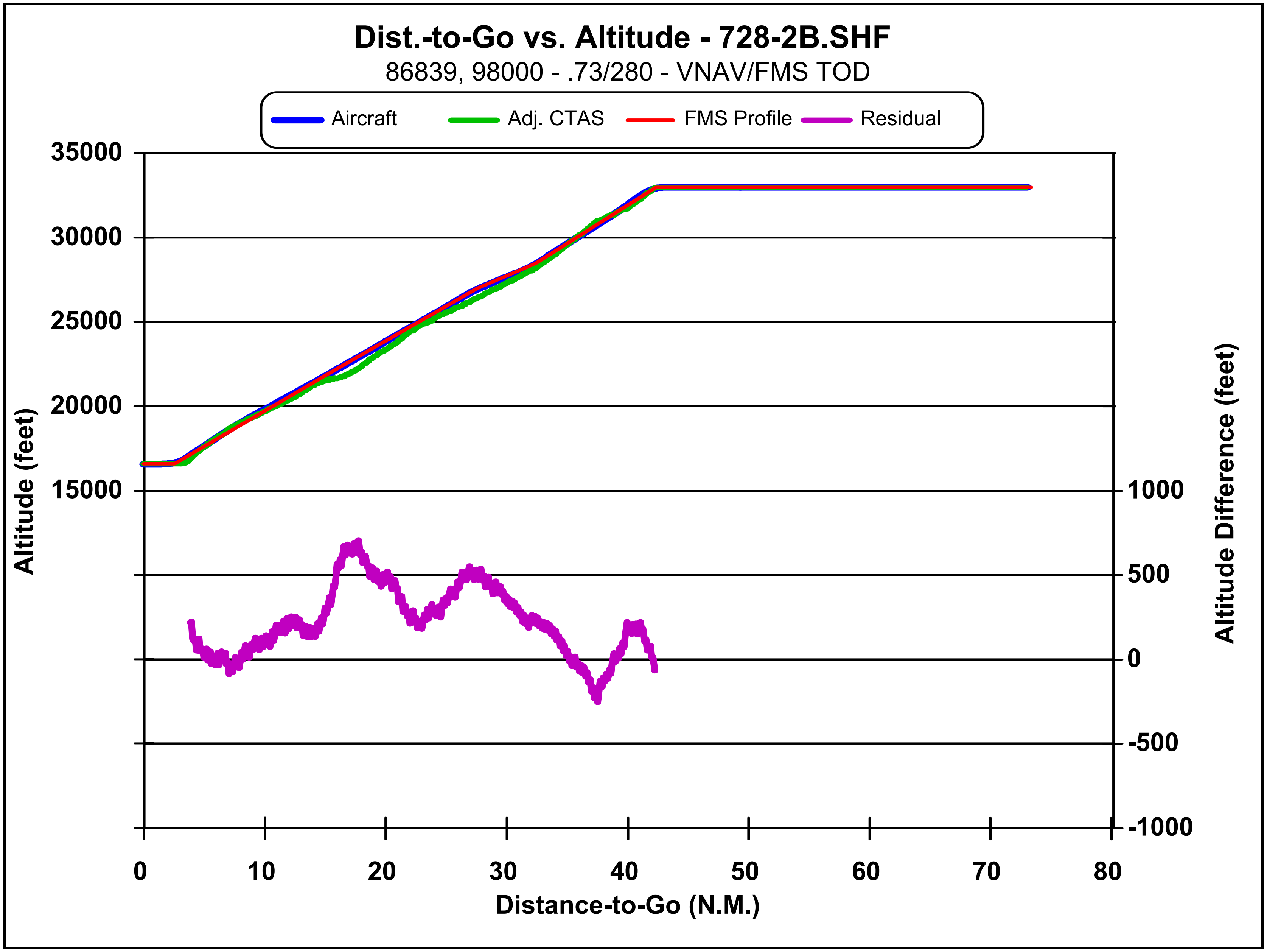
Vertical Descent Initiation Error (Average): 1,859 feet

Residual Error: 222 feet

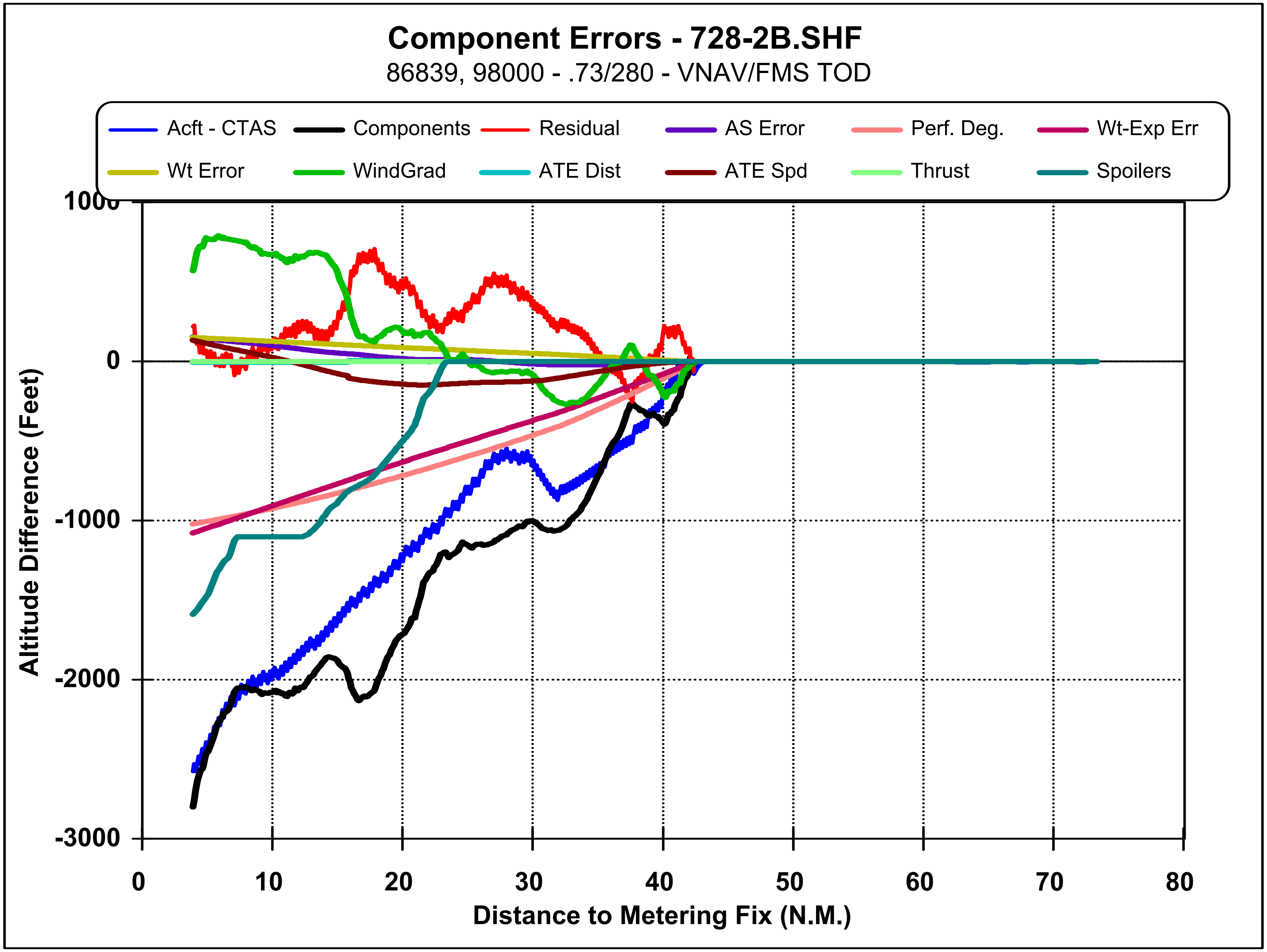

Initial Altitude Parameters
Altitude Error/Residuals - 728-2B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 728-2B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 728-2B.SHF
86839, 98000 - .73/280 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 728-2B.SHF
86839, 98000 - .73/280 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 731-2

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .73/280 KIAS

Aircraft Weight: 88,743 pounds

CTAS Weight: 85,000 pounds

Weight Experimental Error: 0 pounds

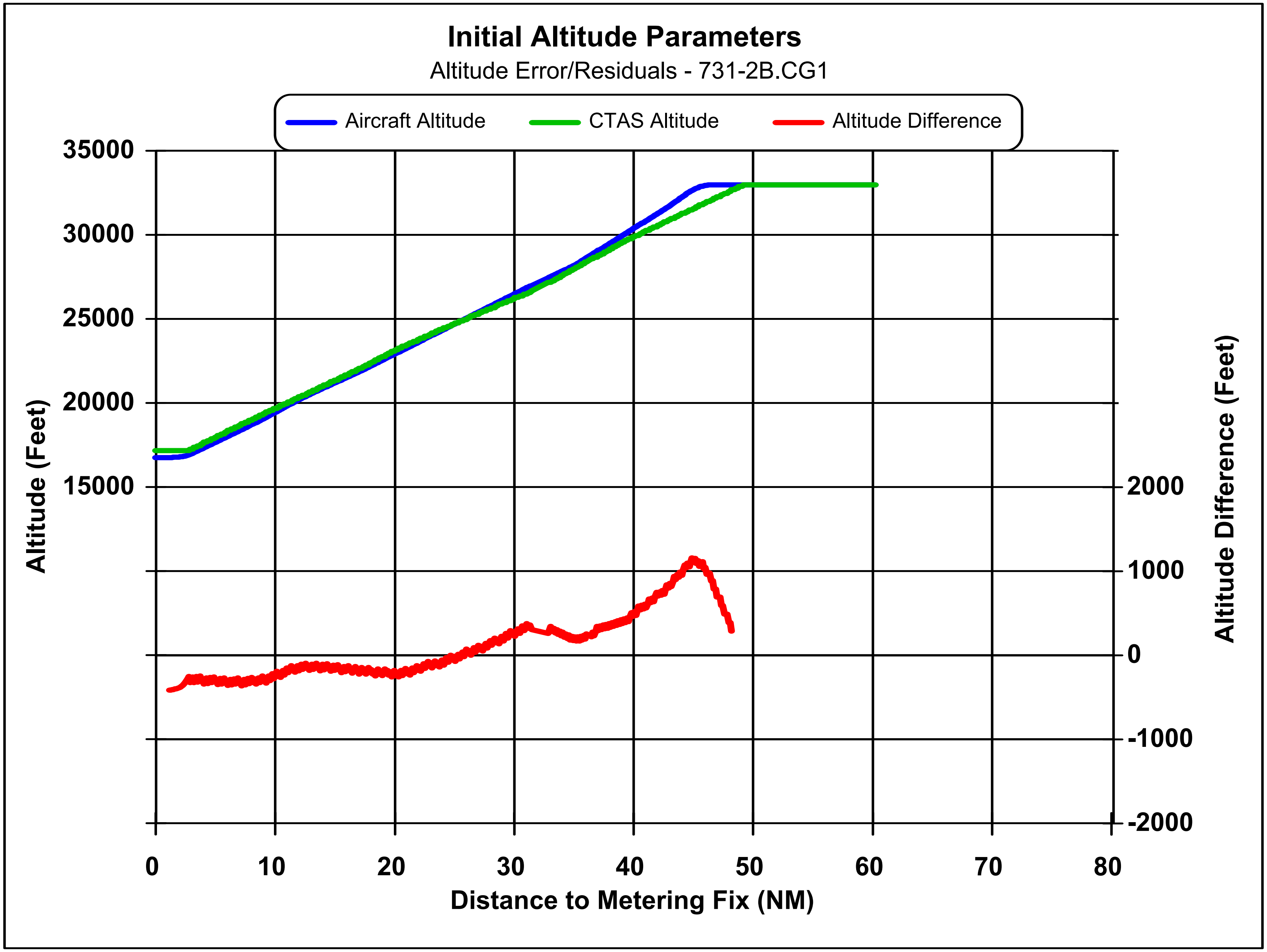
Weight Error: 3,743 pounds

Descent Initiation Error (Time): 27 seconds

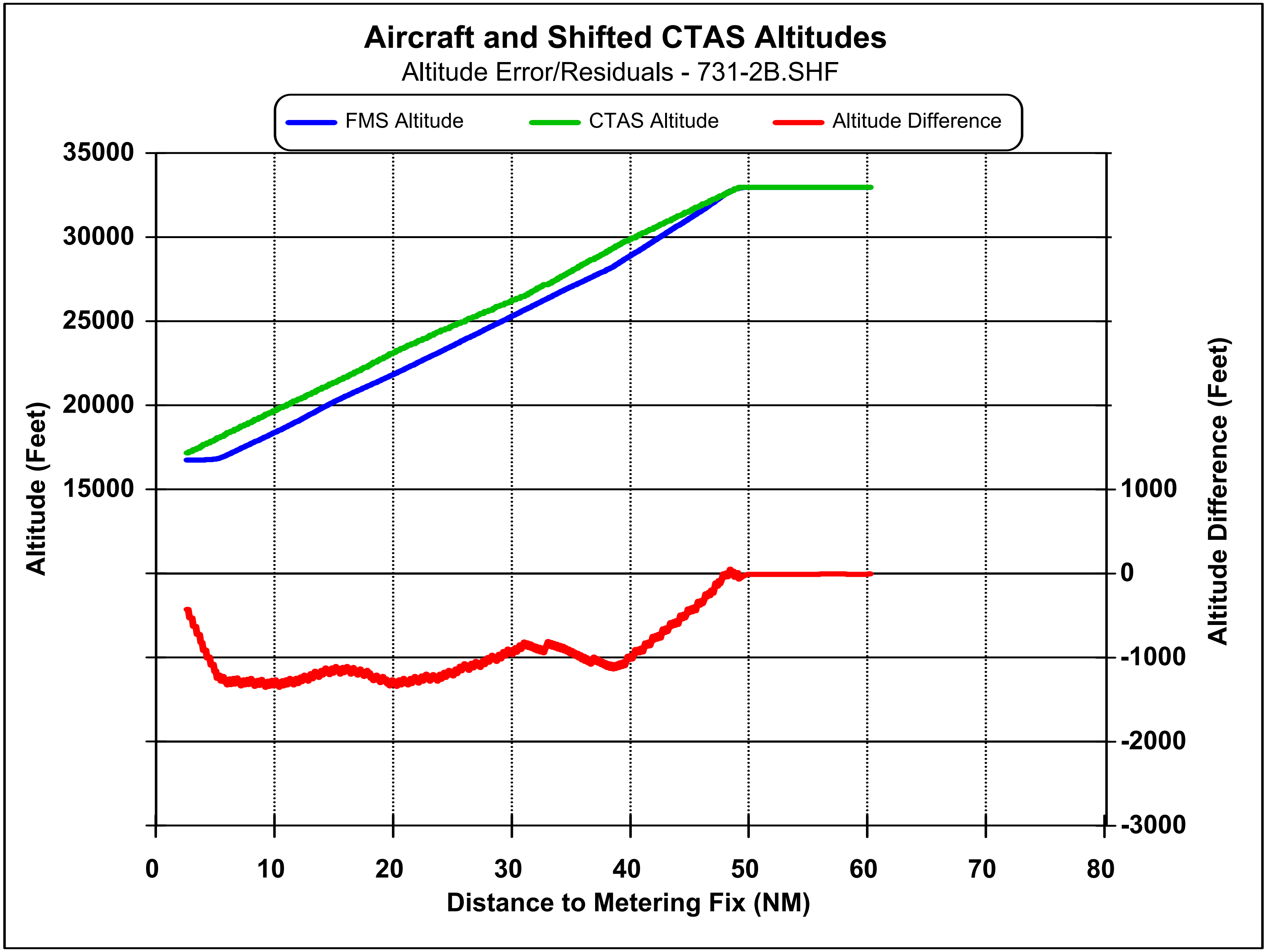
Descent Initiation Error (Distance): 3.350 NM

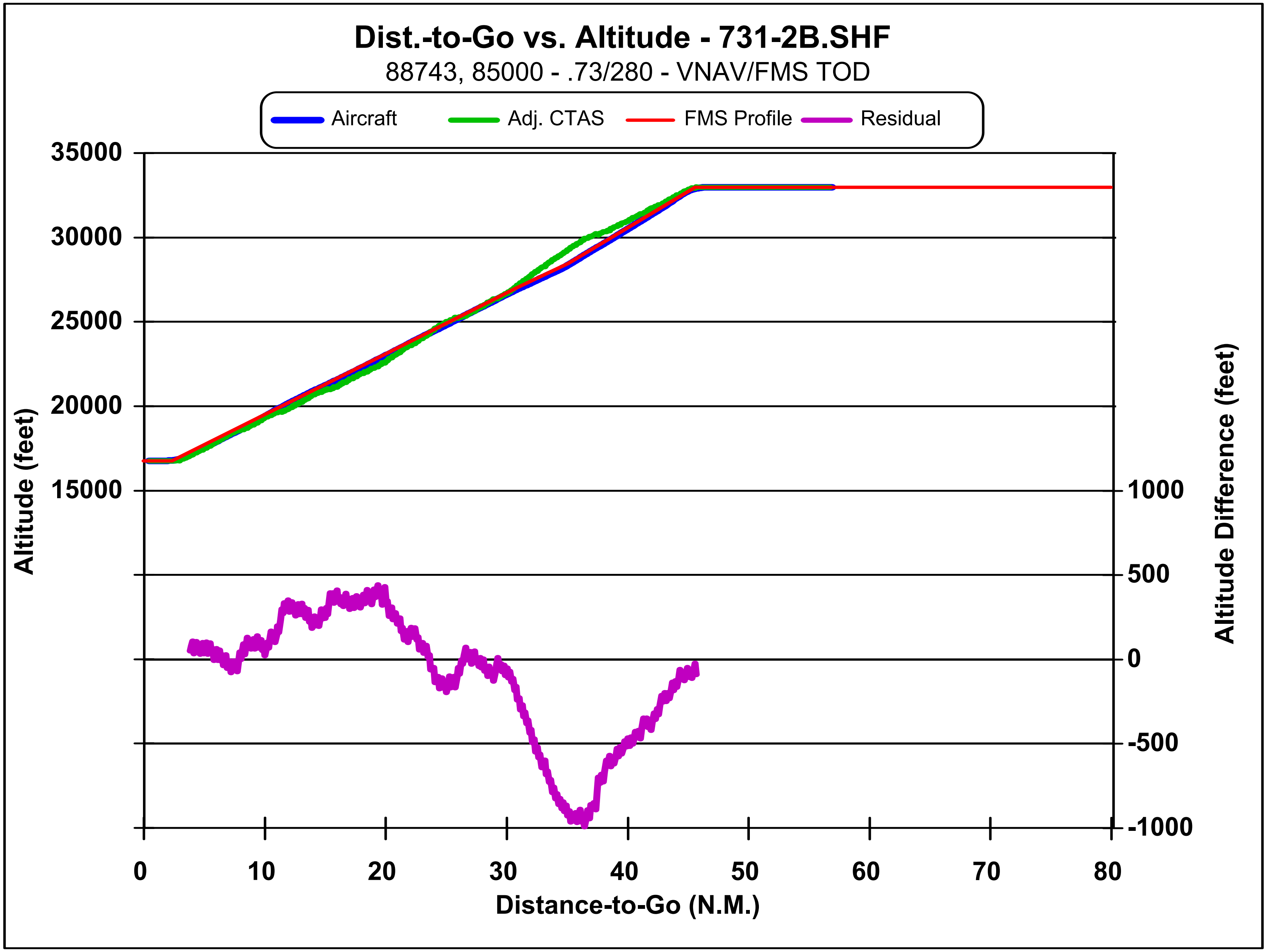
Vertical Descent Initiation Error (Average): 1,004 feet

Residual Error: 53 feet

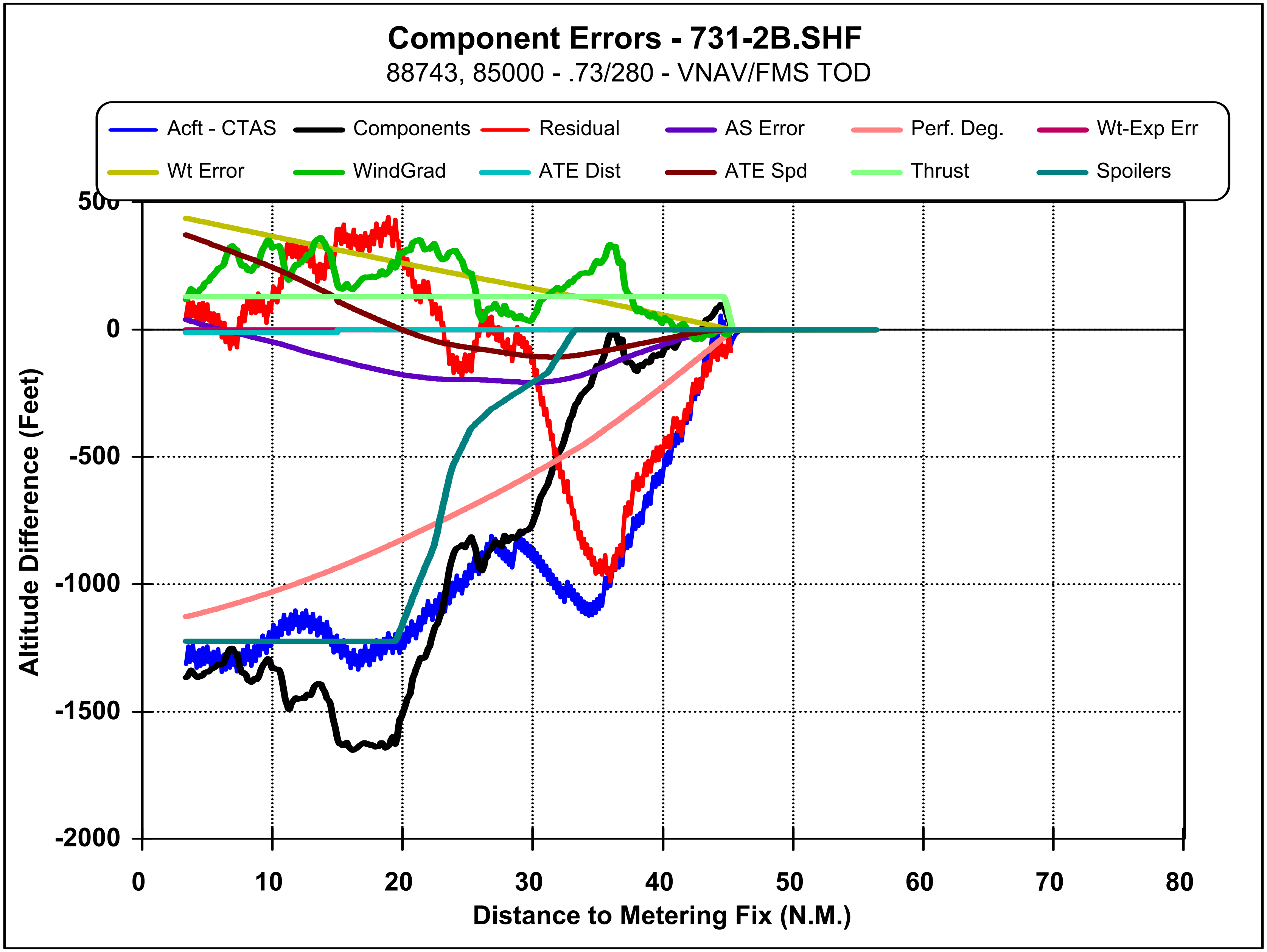


Initial Altitude Parameters
Altitude Error/Residuals - 731-2B.CG1
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 731-2B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
1000
0
-1000
-2000
-3000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 731-2B.SHF
88743, 85000 - .73/280 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)



Component Errors - 731-2B.SHF
88743, 85000 - .73/280 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
500
0
-500
-1000
-1500
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 730-1

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .76/320 KIAS

Aircraft Weight: 92,831 pounds

CTAS Weight: 85,000 pounds

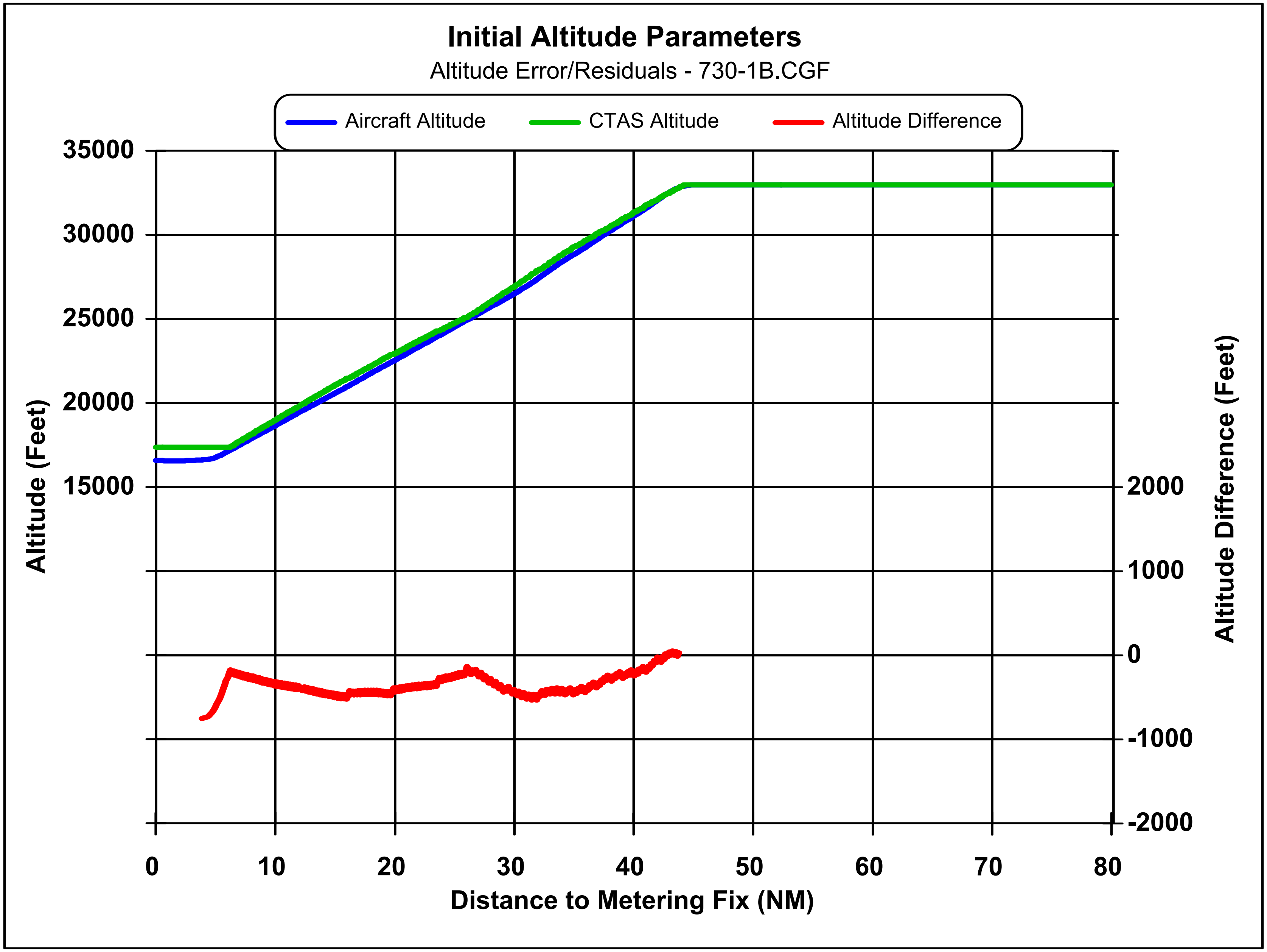
Weight Experimental Error: 0 pounds

Weight Error: 7,831 pounds

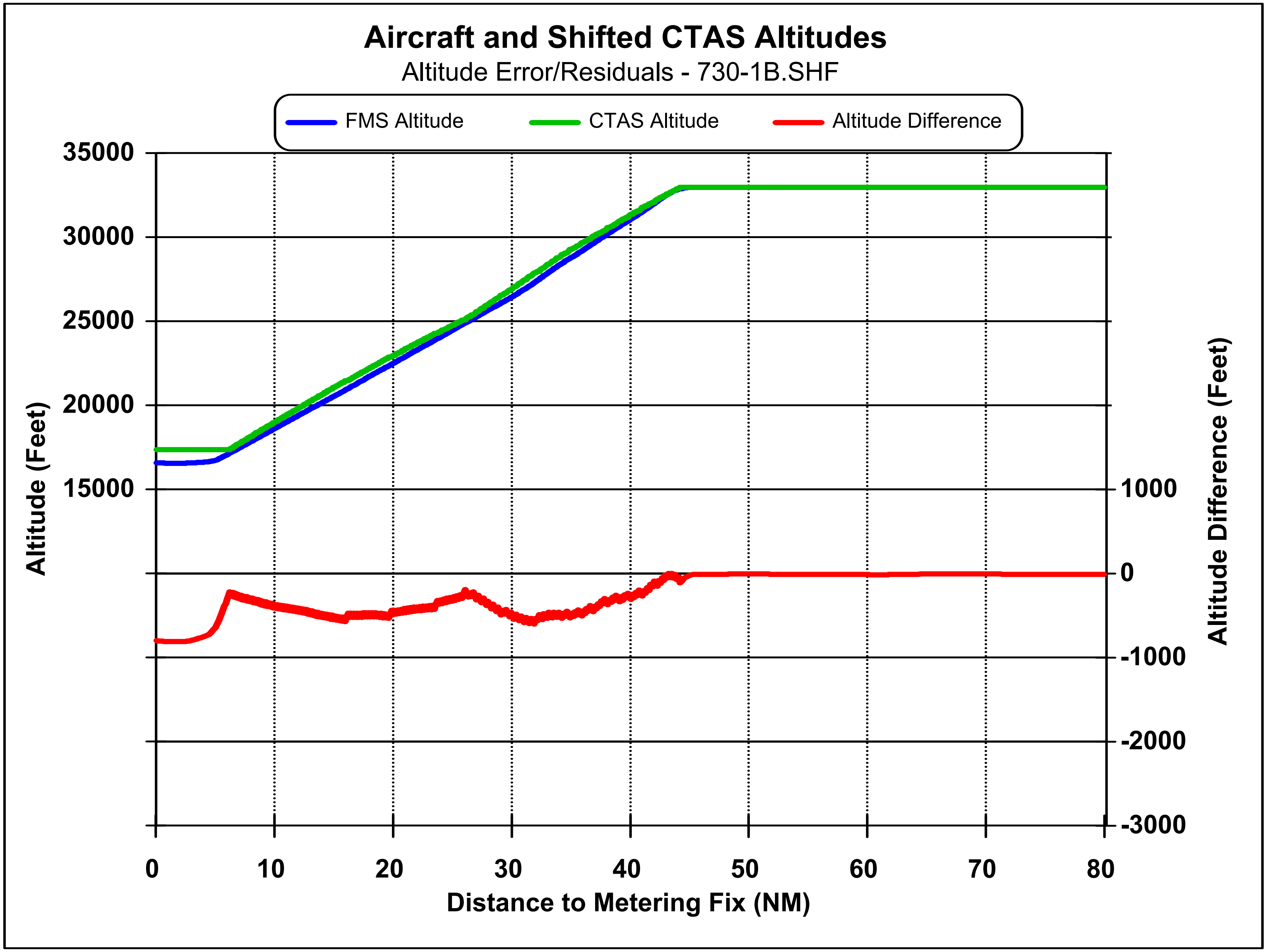
Descent Initiation Error (Time): 1 seconds

Descent Initiation Error (Distance): 0.133 NM

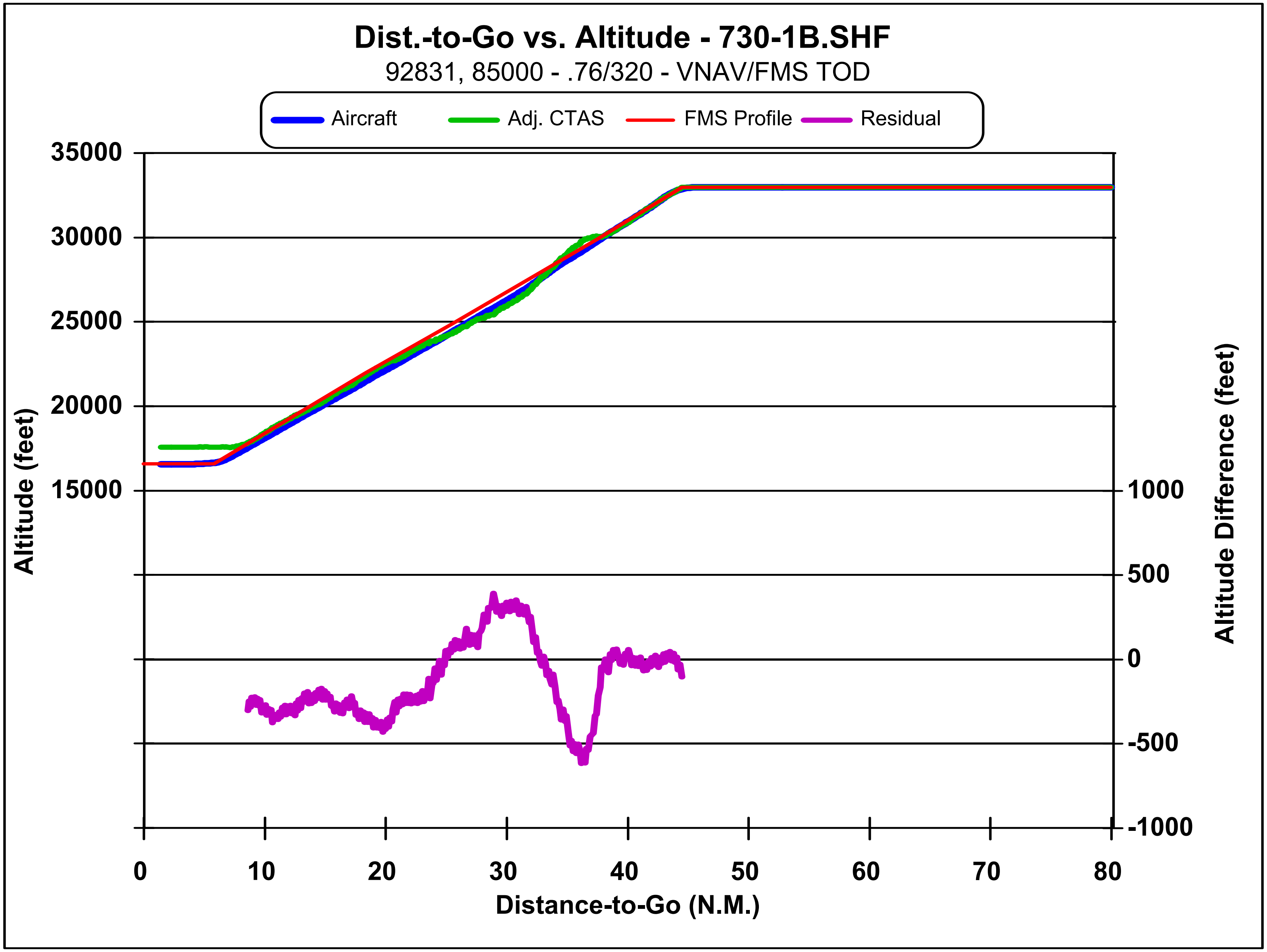
Vertical Descent Initiation Error (Average): 54 feet
Residual Error: -299 feet



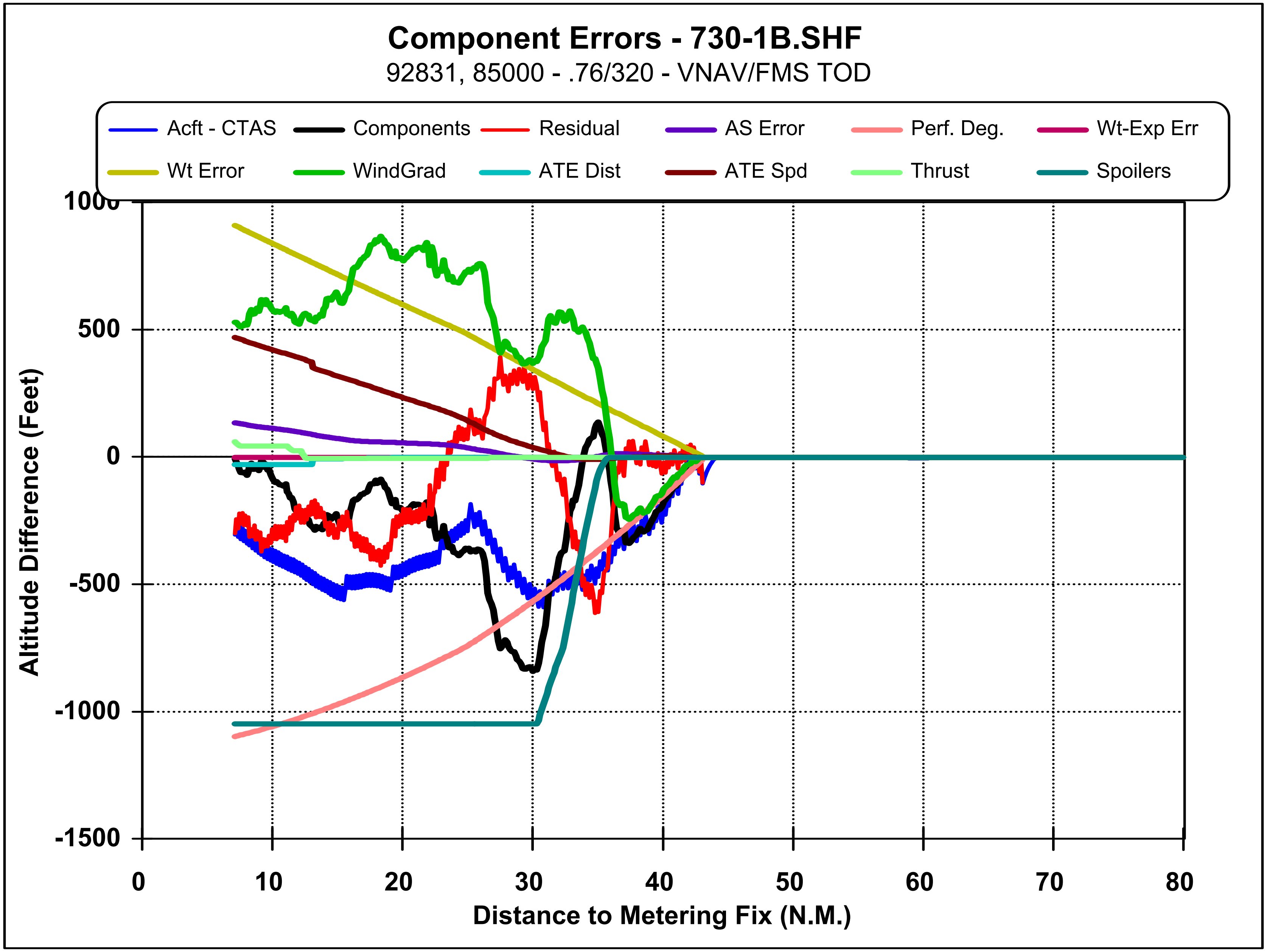
Initial Altitude Parameters
Altitude Error/Residuals - 730-1B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
2000
1000
0
-1000
-2000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 730-1B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
1000
0
-1000
-2000
-3000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Dist.-to-Go vs. Altitude - 730-1B.SHF
92831, 85000 - .76/320 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 730-1B.SHF
92831, 85000 - .76/320 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1000
500
0
-500
-1000
-1500
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)

Run: 732-1

Navigation: VNAV Descent with FMS TOD

Mach/Speed Schedule: .76/320 KIAS

Aircraft Weight: 93,141 pounds

CTAS Weight: 98,000 pounds

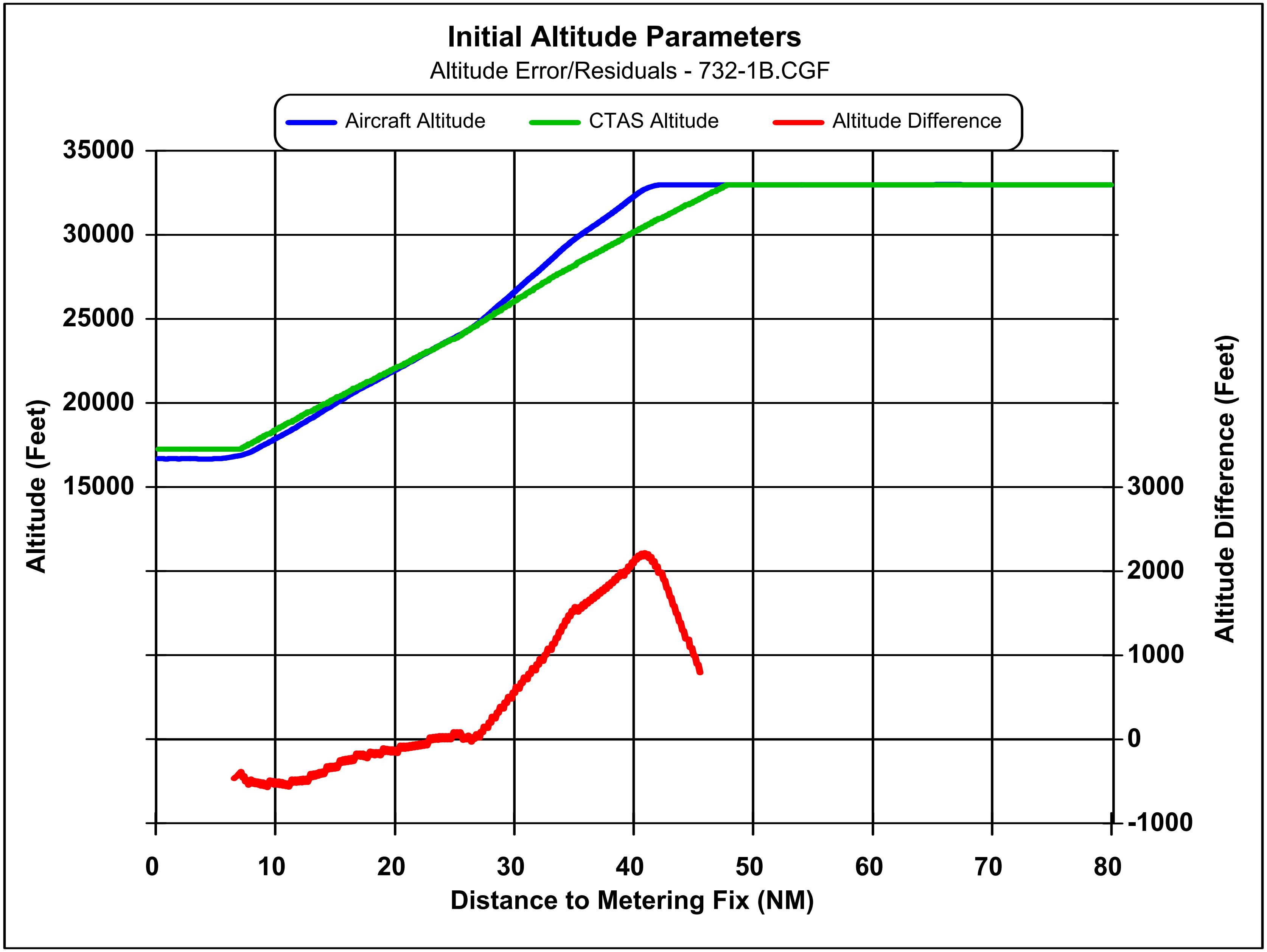
Weight Experimental Error: 13,000 pounds

Weight Error: 8,141 pounds

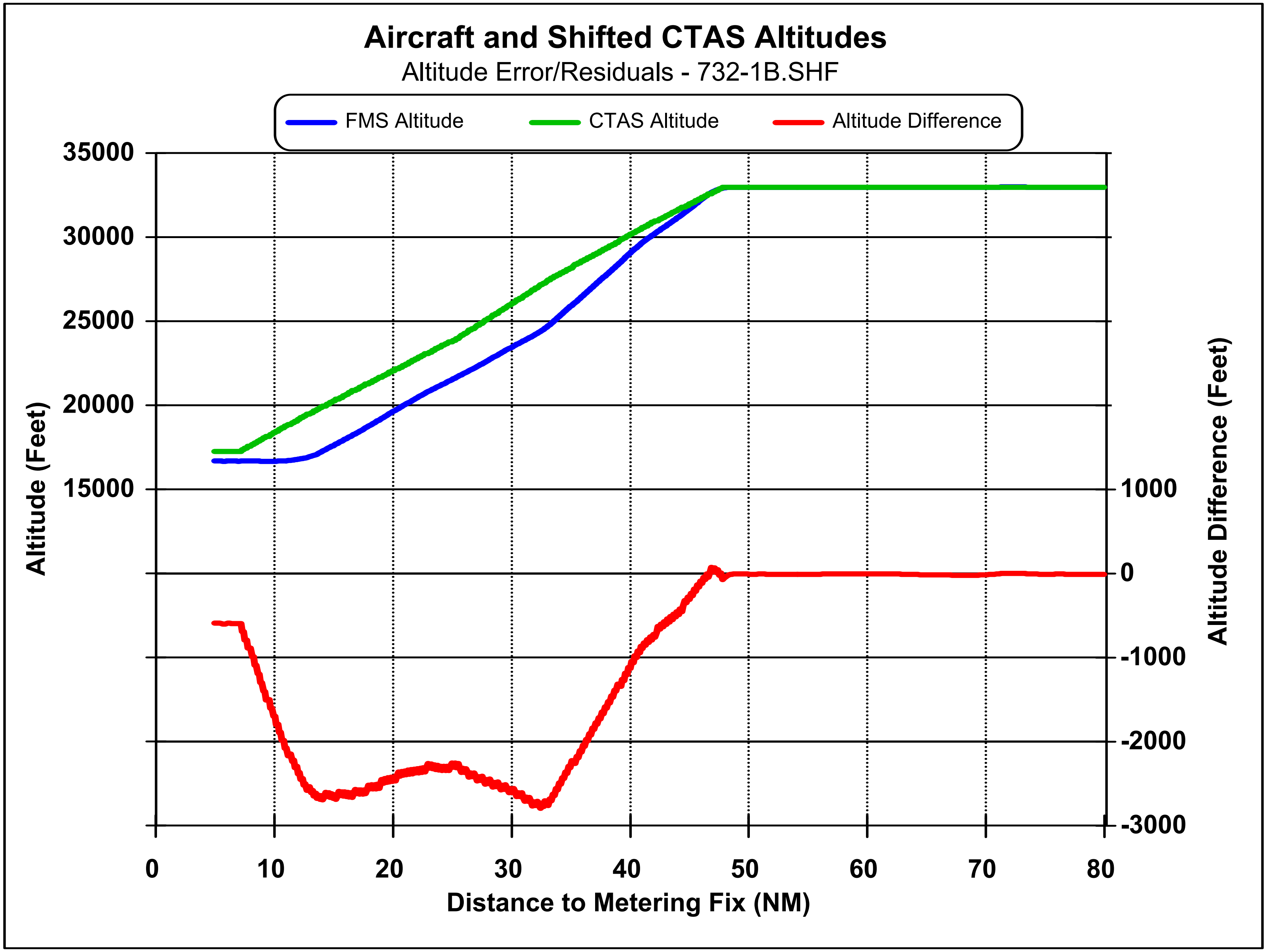
Descent Initiation Error (Time): 46 seconds

Descent Initiation Error (Distance): 5.869 NM

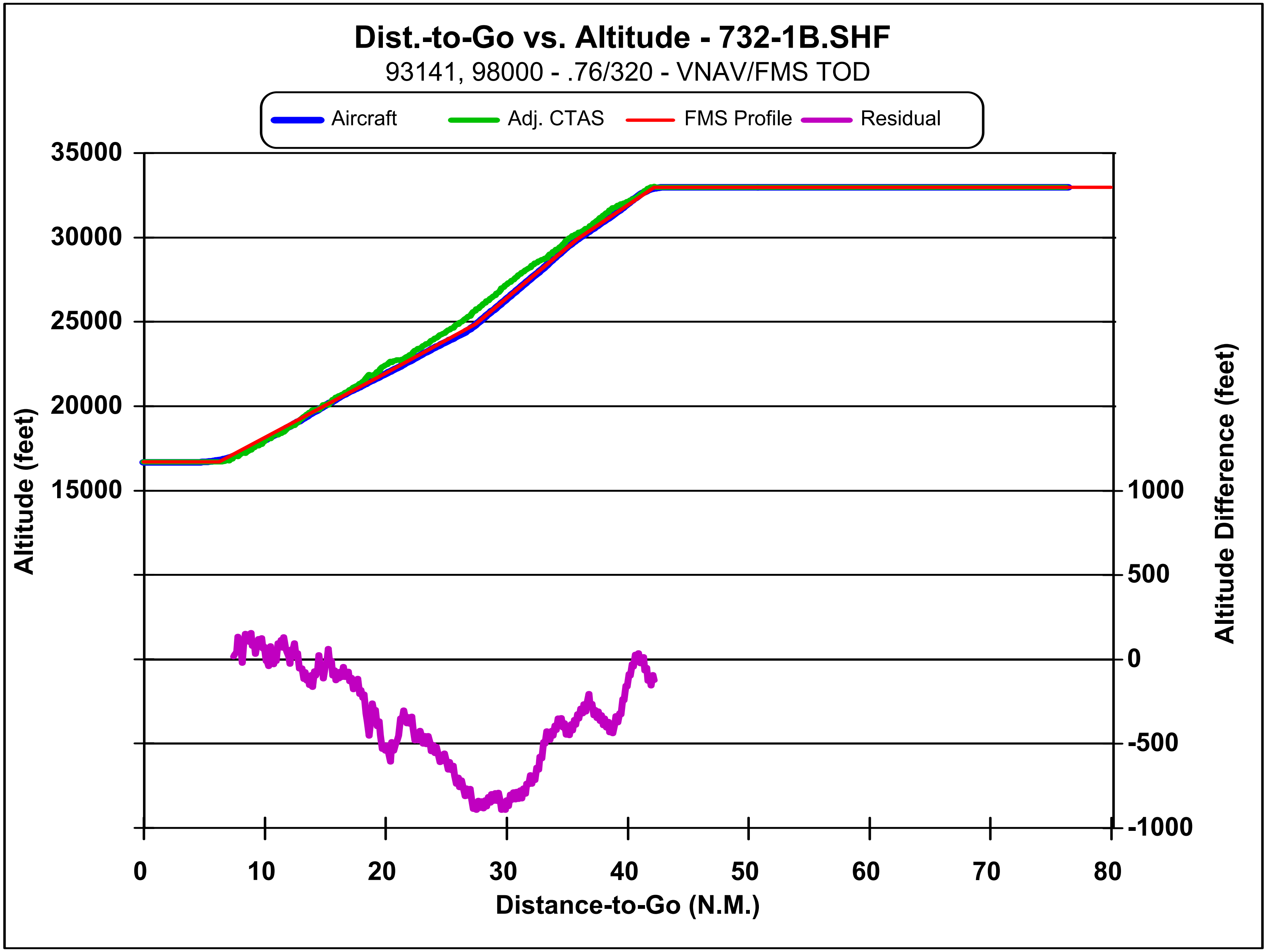
Vertical Descent Initiation Error (Average): 2,012 feet
Residual Error: 20 feet

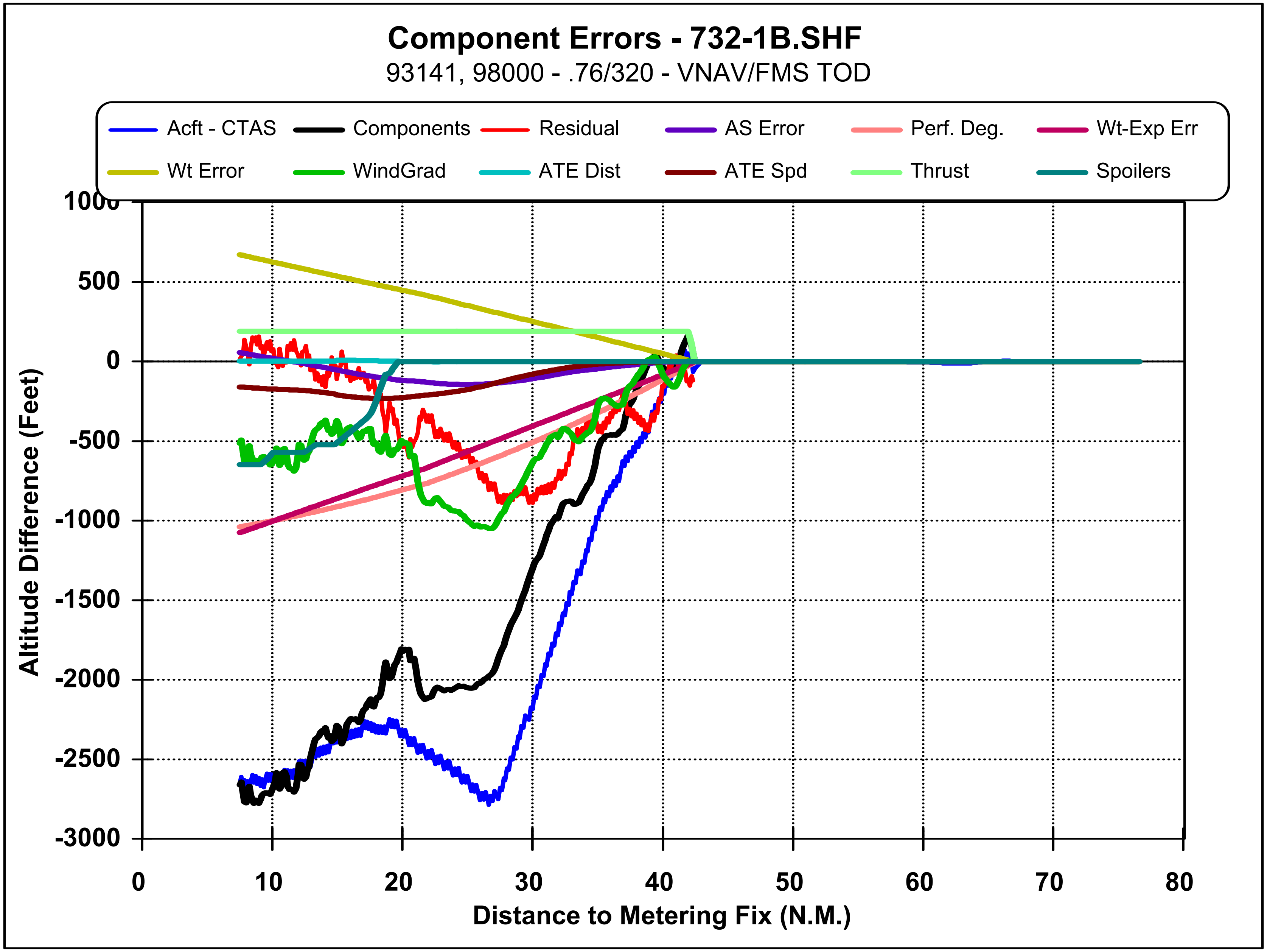


Initial Altitude Parameters
Altitude Error/Residuals - 732-1B.CGF
Aircraft Altitude
CTAS Altitude
Altitude Difference
35000
30000
25000
20000
15000
Altitude (Feet)
3000
2000
1000
0
-1000
Altitude Difference (Feet)
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)



Aircraft and Shifted CTAS Altitudes
Altitude Error/Residuals - 732-1B.SHF
FMS Altitude
CTAS Altitude
Altitude Difference
Altitude (Feet)
35000
30000
25000
20000
15000
Altitude Difference (Feet)
1000
0
-1000
-2000
-3000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (NM)


Dist.-to-Go vs. Altitude - 732-1B.SHF
93141, 98000 - .76/320 - VNAV/FMS TOD
Aircraft
Adj. CTAS
FMS Profile
Residual
35000
30000
25000
20000
15000
Altitude (feet)
1000
500
0
-500
-1000
Altitude Difference (feet)
0
10
20
30
40
50
60
70
80
Distance-to-Go (N.M.)


Component Errors - 732-1B.SHF
93141, 98000 - .76/320 - VNAV/FMS TOD
Acft - CTAS
Components
Residual
AS Error
Perf. Deg.
Wt-Exp Err
Wt Error
WindGrad
ATE Dist
ATE Spd
Thrust
Spoilers
Altitude Difference (Feet)
1000
500
0
-500
-1000
-1500
-2000
-2500
-3000
0
10
20
30
40
50
60
70
80
Distance to Metering Fix (N.M.)